AdoreQS: Quasi-Static Equilibrium Solution for Rolling Bearings

Subset of Parent Program ADORE: Advanced Dynamics of Rolling Elements

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About AdoreQS

AdoreQS is a subset of the parent program, ADORE (1), which is an application to perform a generalized dynamics analysis of rolling bearings. When carrying out a dynamic analysis, the differential equations of motion of all bearing elements are integrated in the time domain to obtain a real-time simulation and true steady-state behavior of the bearing. The integration of differential equations requires initial conditions, which represent a solution at the starting time. A quasi-static analysis is generally used to compute this solution. For this purpose, ADORE contains a quasi-static module, so that equilibrium analysis may be readily performed.

Aside from specification of initial conditions for a dynamic analysis, the quasi-static solution has a notable design significance, since some of the common bearing design parameters, such as fatigue life, bearing stiffness, contact stresses and lubricant film thickness at rolling element to race contacts, are often unaffected by dynamic interactions. For this purpose, the quasi-static and lubricant traction modules from ADORE are extracted and packaged in a standalone application, AdoreQS.

System Requirements

AdoreQS is a command line FORTRAN application and it is available in executable form for both Windows and Macintosh platforms. The maximum memory requirement is no more than 8 MB and disk storage requirement is less than 10 MB. In addition to the application files, the disk storage is used by two ASCII text files, DATA.txt and PRINT.txt, for program input and output respectively. The accompanying input facility, AdrQsInput, for interactive preparation of the input data file, DATA.txt, is a Java application and it requires the runtime Java Environment, which is normally available on most computer platforms. The Java runtime environment may also be freely downloaded from the oracle website, https://www.java.com/en/download/.

Software installation

In order to maintain system independence, there is no installation or setup file supplied with the software. The software must, therefore, be manually installed. Following are the installation steps:

1. Under any parent folder create a subfolder AdoreQS.
2. Download the software file AdoreQS850_Windows.zip or AdoreQS850_Mac.zip respectively for the Windows or Macintosh platforms from www.PradeepKGuptaInc.com/AdoreQS.html.
3. Unzip the downloaded zip file in the subfolder AdoreQS
4. Create a subfolder /bin inside the AdoreQS folder to store the executable files, AdoreQS850.exe and AdrQsInput.jar.
5. Move the executable file AdoreQS850.exe (Windows) or AdoreQS850 exec file (Macintosh), and the java input facility AdrQsInput.jar to the executable subfolder …/AdoreQS/bin.
6. Since the java application, AdrQSInput.jar, runs in an interactive graphic environment a shortcut may be installed on the desktop on a Windows system, and in the dock on the Macintosh platform. To create a shortcut on a Windows machine right click the application icon and select “Create shortcut”. On a Macintosh system simply drag the application icon to the dock.
7. On a Windows platform, set the PATH variable in the environmental variables to include the subfolder …/AdoreQS/bin. The procedure may slightly vary between different versions of Windows. On Windows 10 do the following:
   a. On the Control Panel Screen, select “System and Security”.
   b. On the next screen select “System”.
   c. On the next screen select “Advanced system settings” in the left margin.
   d. In the next window click on “Environmental Variables” button.
   e. In the next window, under User Variables, select “Path” and click on “Edit” button.
   f. In the next window click on the “New” button.
   g. In the main window type the full path to the /bin folder, for example:
      c:\Users\....\Documents|AdoreQS\bin
   h. Click on the OK button in the three open windows and close the System window.
8. On Macintosh platform, the PATH variable is set under “Preferences” in the “Terminal” application, which is found in the Utilities subfolder in the Applications folder. For convenience a short cut may be installed in the dock, by dragging the application icon to the dock. To set the path variable do the following:
   a. Open the “Terminal” application and Click on the “Terminal” tab in the menu bar and select “Preferences.
   b. In the next window click on the “Shell” tab.
   c. Now under Startup, turn on the “Run Command” check box and type the full path to the /bin folder, for example: PATH=$PATH:/Users/…./Documents/AdoreQS/bin.
   d. Close the window by clicking on the red button in the top left corner.
   e. Quit and restart the Terminal application to activate the new PATH variable.

Software installation is now complete.

Software Execution and Testing

Both the java application AdrQSInput and the FORTRAN application AdoreQS must be executed in order to confirm proper installation and operation of the applications.

Command Prompt: Since AdoreQS is a command line application access to command prompt window is essential on the available computer system. On the Windows platform the command prompt application is generally available in the accessories folder. On Windows 10 it may be easily found by doing a search on “Command Prompt”. For convenience a short cut to this application may be installed on the desktop. On the Macintosh operating system, the command line tool is called “Terminal” and it is available in the Utilities folder under the Applications folder. Drag the application in the dock to install a short cut for convenience.

Java Application AdrQsInput: If the Java runtime facility is installed on the available computer platform a double click either on the AdrQsInput shortcut or the application icon should bring up the application. If this does not work, try the following command in the command prompt window after navigating to the folder which contains AdrQsInput850.jar:

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java -jar AdrQsInput850.jar
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If this turns out to be an invalid command, then java runtime facility is not installed on the system. Install it from the oracle website and try again.

If the application runs successfully, the AdrQsInput window will open along with the program terms and conditions window. Read and the terms and conditions, and indicate your acceptance by clicking the “OK” button in that window. The application is now running successfully.

A typical data screen is shown in figure 1. By default, the data screens are populated for typical rolling bearing, used in the generating the test cases supplied with the software. All the data items are listed on the left and explanation on the variables is documented on the right in a scrollable window. When preparing the data file first time, the data fields may be interactively edited to change the data to desired values. When all data values on the screen are updated, click the “Next Rec” button for the next data screen. To go back to the previous screen, click the “Go Back” button.

When the last data record is reached, a message window will appear to indicate so. Click “OK” in this window and save the data file using one of the following two options:

1. When creating the data first time, click the “File” tab on the menu bar and click on “Save As..”. This will open a file navigation window. Navigate to the desired folder, assign a name to the data file and click the “Save” button. The default input data file name used in AdoreQS is DATA.txt. Therefore, it is convenient to save the file with this name. Alternatively, any name may be used and before running AdoreQS, the file may be renamed to DATA.txt.
2. The “Save & Exit” button may be used when editing an existing data file. The original file is over written in this case.

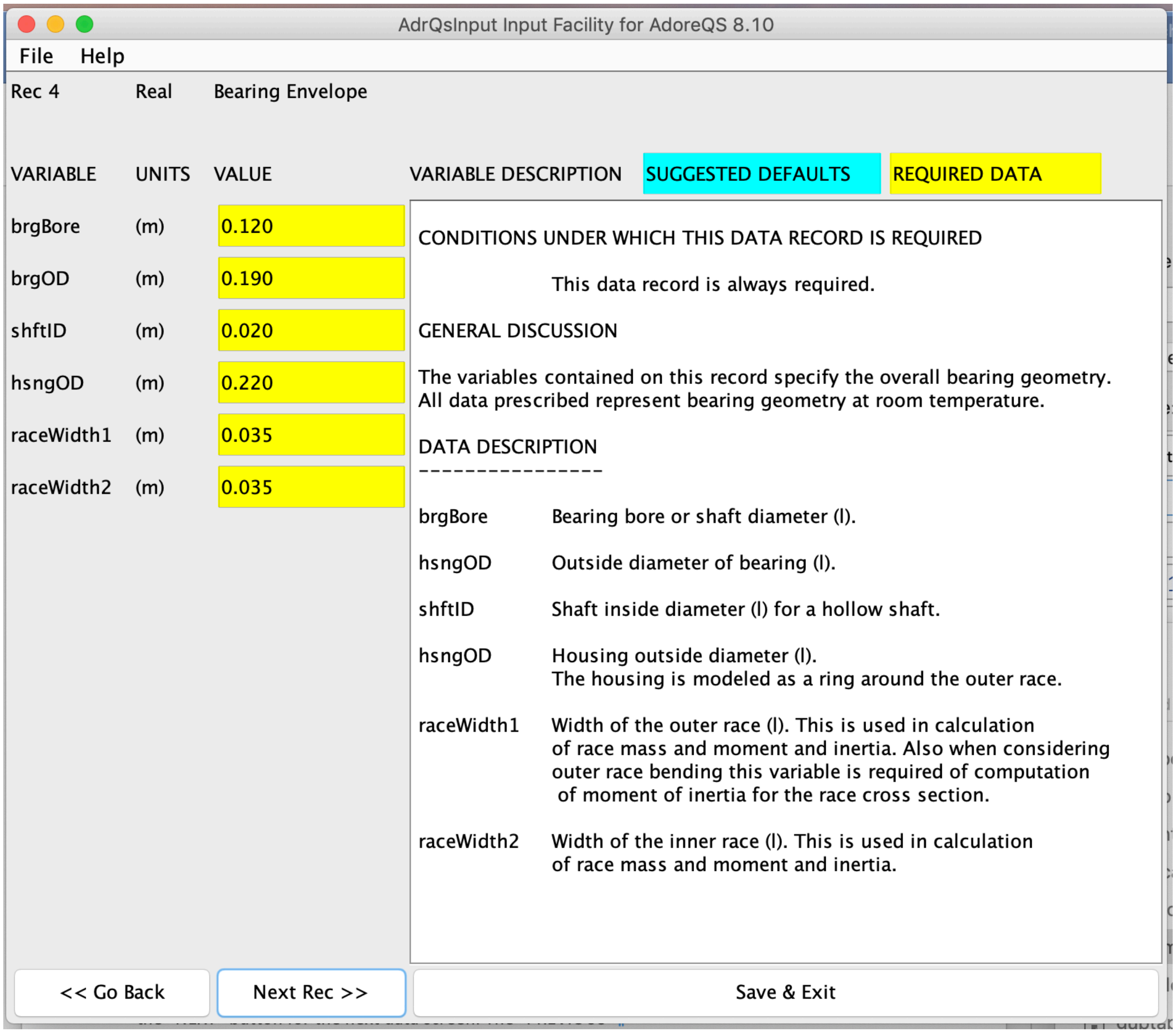

Figure 1. Typical AdrQSInput Data Screen.

To open an existing data file, click on the “File” tab on the menu bar and select the “Open” option. This opens a file navigation window. Navigate to an existing data file and click “Open”. This will load the existing data in the application and the data may now be edited interactively.

The above feature may be experimented with one of the test data files supplied with the application. To do this, do the following:

1. Start the java application AdrQsInput.
2. Click of the “File” tab in the menu bar and select “Open”.
3. Now navigate to the file, ../AdoreQS/TestCases/Ball_Bearing/DATA.txt and click “Open”
4. The above steps will open an existing data file and load all the data in the application. The data may now be edited. For now, do not make any changes and click on the “File” tab and select “Quit”. A message window that any changes made to the data will be destroyed is posted. Click “OK” to exit the application.

The “Help” tab in the menu bar provides information on a few common items. The data record numbers and variables names on the various data records, conform to those used in parent program ADORE. Therefore, the ADORE User Manual may be used to get more detailed information about the data items. This manual may be found at:

http://www.pradeepkguptainc.com/Documents/adoreManual.pdf

Running AdoreQS: Four test cases for ball, cylindrical roller, spherical roller and tapered roller bearings are supplied in the TestCases subfolder. In order to validate software installation and operation, one or all of these test cases may be executed. The results should compare with those supplied in the corresponding PRINT.txt output files for the test cases. To make a typical run, do the following:

1. Create any subfolder ../Test anywhere on the available disk drive.
2. Copy the input data file for one of the test cases, for example, …/AdoreQS/TestCases/Ball_Bearing/DATA.txt, to the current Test subfolder.
3. Go to the command prompt window on Windows system or open the Terminal application on Macintosh system, and navigate to the current Test subfolder.
4. Execute the command “DIR” on Windows or “ls” on Macintosh to get the list of files in the current folder. The input data file “DATA.txt” should be there in the subfolder. Existence of the input data file is essential for the application to run successfully.
5. Now invoke the command “AdoreQS850”. This will start the AdoreQS application. Press “Enter” to accept the terms and conditions and continue with execution of the application.

Based on input data and the computing speed of the available system, there may be a pause for a few minutes. When the execution is completed a message indicating so will be displayed. The program output may now be viewed by opening the file “PRINT.txt” is the ../Test subfolder.

Most of the output is quite self-explanatory. Again, the output variable names conform to those used in the parent program, ADORE. Therefore, the Adore User Manual, available at the link, presented above, may be used to get more details.

AdoreQS Capabilities

For practical design and performance diagnosis following are the capabilities of AdoreQS:

- Types of bearings include ball, cylindrical roller, spherical roller and tapered roller bearings. However, the number of rows of rolling elements in limited to a single row. Therefore, spherical roller bearings are modeled in terms of individual rows using appropriate constraints. See the spherical roller bearing example in the TestCases folder.
- In angular contact ball bearings, the races may be split. Both three and four point contacts may be modeled.
- Geometrical imperfections on ball in a ball bearing are limited to ball size variation which may either be arbitrarily prescribed or randomly distributed.
- In roller bearings, geometrical imperfections may be prescribed on each dimension of the roller.

- Geometrical imperfection on bearing races may either be simulated by an elliptical profile or by a sinusoidal variation. In ball bearings both the race radius and groove curvature may be varied.
- Common bearing materials are available in the materials data base built in the application, and they may be easily selected by using appropriate materials code. In addition, arbitrary material properties may also be prescribed in the input data. For certain bearing materials the variation of properties as a function of temperature is included in the data base. This permits automatic selection of applicable properties at prescribed temperature.
- Either load or relative displacement may be prescribed on the bearing to simulate a loaded bearing. Likewise, either moment or misalignment may be prescribed.
- Angular velocities are computed by kinematic considerations. For angular contact ball bearings, the empirical kinematic hypotheses include, conventional race control, the newly developed minimum energy hypothesis or prescribed orientation of ball angular velocity vector.
- A number of commonly used lubricants are included in the lubricants data base. In addition, arbitrary lubricant may be user specified.
- Bearing races may be subject to interference fits.
- Centrifugal and thermal expansion of bearing races is modeled under prescribed operating environment.
- Operating race fits and internal clearance in the bearing are included in the program output.
- Individual contact solutions include contact loads, stresses, elastic deflection and contact geometry.
- Computation of rolling element to race slip, and the resulting frictional dissipation and wear, are subject to input kinematic constraint.
- Either one or both races may be subject to rotation.
- Bearing stiffness is computed by multiple static equilibrium solutions.
- AdoreQS includes fairly comprehensive modeling bearing fatigue life. The key features of the life models include the following:
    - Independent modeling of life of bearing races and rolling elements.
    - Life equations include distinct geometrical and materials parameters so that life of modern hybrid bearings may be more precisely modeled.
    - The role of residual stresses as applicable in case hardened materials is modeled as appropriate failure stress modification.
    - The applicable failure stresses are also modified as a result of hoop stresses generated as a result of shrink fits and thermal and centrifugal expansion of bearing races.
    - Appropriate life modification factors for advanced materials, manufacturing processes and lubrication effects are implemented to modify the basic life.

Limitations

- Analytical foundation of AdoreQS is limited to static equilibrium.
- Number of rolling elements in the bearing is presently limited to 40. However, this may be easily changed if needed.

- All bearings are limited to one row of rolling elements. Therefore, treatment of spherical roller bearings must be segmented into individual rows. See the spherical roller bearing test case.
- All bearings are cageless, since any reasonable cage analysis requires a truly dynamic analysis.
- Lubricant churning and drag effects are not included in AdoreQS.
- Computation of bearing heat generation and wear is subject to the input kinematic hypothesis for computation of rolling element velocities and resulting slip rates.
- All solutions are obtained under prescribed operating temperatures and AdoreQS does not carry out any thermal analysis.

Technical Background

As shown schematically in figure 2, modeling performance of any mechanical component, such as a bearing, gear or seal, essentially consists of an integration of three fundamental elements:

1. Material behavior or constitutive equations.
2. Geometric compatibility which includes operating conditions and any constraints.
3. Governing equations, which define the operation of the component being modeled.

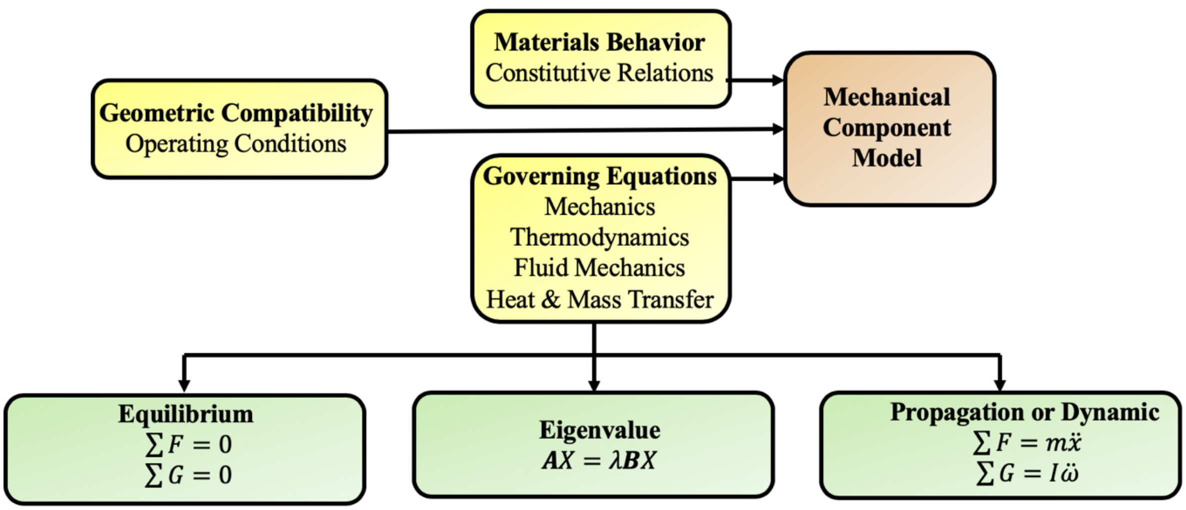

Figure 2. Schematic layout of the fundamental elements of a model for performance simulation of mechanical component.

For rolling bearings, the materials section includes all materials, including the lubricant, used in the bearing. Any constraints, such as restricted displacements, bearing type and geometry, and the operating conditions, such as applied loads and speeds are included in the geometric compatibility section. The governing equations may be broadly classified into three types:

1. Static Equilibrium: Here all the interacting forces and moments are summed up to zero to determine an equilibrium solution.

2. Eigenvalue: This is also a type of equilibrium solution but under a critical parameter (λ). Computation of natural frequency of vibration or critical speed of a rotor are typical examples.
3. Propagation or Dynamic: Modeling time dependent behavior or transient conditions constitutes a propagation or a dynamic problem. Here motion of the elements in the component being modeled are defined by differential equations of motion, which are integrated in time domain to obtain the transient and steady-state performance simulations.

While all three types of analyses may be performed in ADORE, AdoreQS is restricted to static equilibrium.

Aside from a few specialized bearing applications involving noise and vibration analysis, static equilibrium and a dynamic formulation are the two most common types of analyses used to model rolling bearing performance. Since the centrifugal forces acting on the rolling elements affect the contact forces between the rolling elements and the races, these forces are included as additional applied forces in the equilibrium analysis. The static models are therefore, referred to as quasi-static, although the underlying formulation remains an equilibrium formulation. As discussed above, AdoreQS is essentially the static equilibrium module of ADORE. However, before presenting the equilibrium model in more detail, a brief comparison of the static and dynamic models in terms of the differences in formulation and practical significance may be useful.

Basic differences between the static and dynamic models for rolling bearings are outlined in table 1. Note that while, time-varying operating conditions, such as variation in applied loads and/or operating speed, clearly constitute a dynamic condition, there is a lot of dynamics in the bearing even under constant load and speed. For example, in a ball bearing with cage or separator, the balls constantly collide in the cage pockets and generate impact forces. While these forces are quite small in comparison to the applied contact forces at the ball-to-race contacts, these collision forces are the only forces acting on the cage, therefore, cage motion is truly dynamic and a realistic simulation requires formulation of the differential equations of motion. Likewise, when the balls are skidding, rather than rolling at a constant speed, or the rollers in a roller bearing are skewing, i.e., oscillating about the transverse axis, the motion becomes dynamic. Under normal steady operating conditions, however, the behavior at the rolling element to race contacts is unaffected by subtle cage forces and a static equilibrium solution may provide a lot of information related to behavior of the bearing. Thus, static equilibrium formulations are widely used in the industry for conventional rolling bearing design.

In terms of practical significance, the static model may be adequate when the interest is in the following characteristics:

- Overall load distribution
- Contact stresses
- Nominal lubricant film thickness
- Fatigue life
- Bearing Stiffness

A dynamic model is required when time dependent bearing interactions need to be modeled. Typical bearing interactions in this category include:

- Cage motion and instability
- Rolling element skid
- Roller skew
- Lubrication effects, such as instabilities related to lubricant traction
- Wear modeling
- Heat generation
- Bearing torque variation
- Dynamic or time-varying loads and speeds
- Irregular bearing geometry
- Optimization of manufacturing tolerances
- Bearing noise and vibrational characteristics

Table 1. Differences between Quasi-Static and Dynamic Models

Quasi-Static Model	**Dynamic Model**
Algebraic equations of equilibrium	Differential equations of motion
Race control / kinematic hypothesis	No such constraint
All velocities are constant	Arbitrary accelerations
Fixed interactions	Interactions vary with time
Restricted treatment of skid & skew	Real time simulation of all motions
No treatment of cage instability	Real time simulation of cage motion
Fixed applied loads	Loads may vary with time
One solution contains all parameters	Time transient solutions

With the above practical significance, it is clear that the static models do provide significant potential in bearing design. Hopefully, AdoreQS serves such a purpose. The objective of the following review of the analytical foundation and model formulation is to provide the user with adequate technical insight for effective use of this software for practical applications.

Quasi-Static Model Formulation

Figure 3 provides a schematic description of a typical ball/race contact. The fundamental variables are relative positions of the ball and race. For prescribed positions the geometric interaction between the ball and race, and the Hertz point contact theory are used to compute the contact loads, and the equilibrium equations may be written as follows:

Ball Equilibrium: For prescribed angular position, θ, of the ball, the axial and radial equilibrium equations are written as:

Axial equilibrium: $\sum_{j=1}^{2} Q_j \sin\alpha_j = 0$ [1a]

Radial equilibrium: $\sum_{j=1}^{2} Q_j \cos\alpha_j - F_c = 0$ [1b]

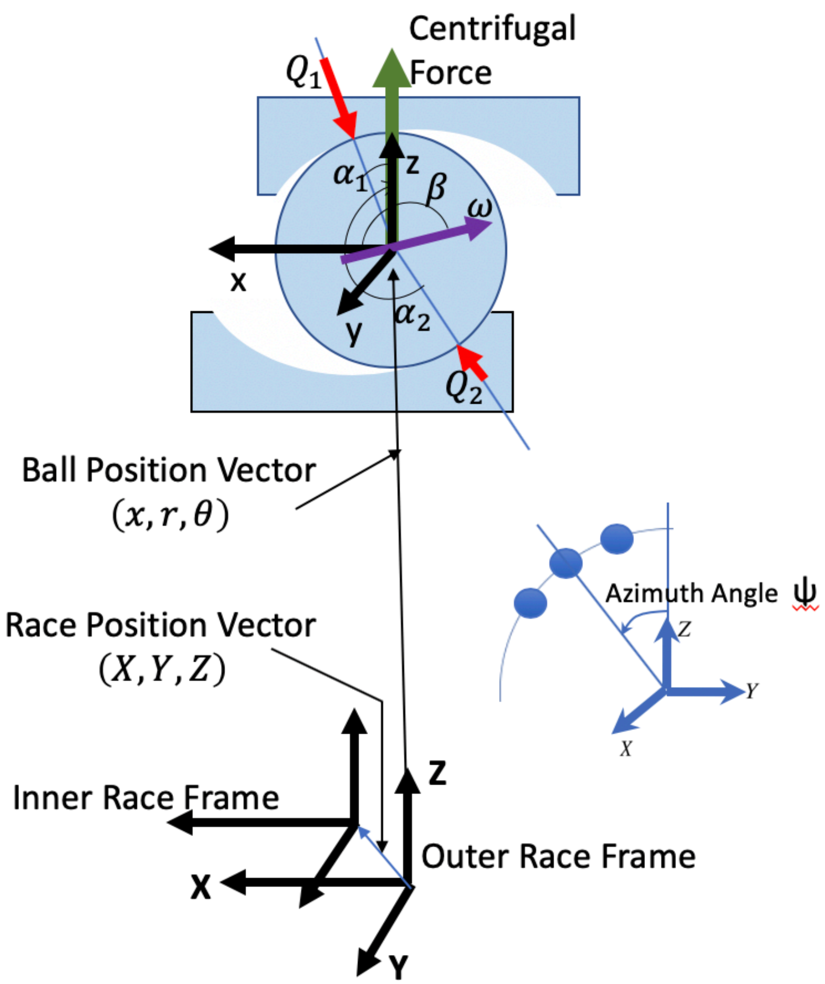

Figure 3. Contact schematic of a typical ball/race contact in an angular contact ball bearing.

Here Q and α are respectively the contact load and angle, and F_c is the centrifugal force on the ball. The subscript j is used to denote outer and inner race with the values of 1 and 2 respectively. Since the contact loads are related to the displacements, the two algebraic equations may be solved to compute the axial and radial position of the ball relative to the race position.

Race Equilibrium: Since it is the relative position, which matters, the outer race may be fixed at the origin of a space fixed coordinate system and the inner race may be displaced relative to this coordinate frame by a vector (X, Y, Z) , as shown schematically in figure 3. Under a prescribed load vector. (Q_x, Q_y, Q_z), the equilibrium equations of the inner race may be written as:

Equilibrium along X axis: $\sum_{i=1}^{n} Q_{2i} \sin\alpha_{2i} = Q_x$ [2a]

Equilibrium along Y axis: $\sum_{i=1}^{n} Q_{2i} \cos\alpha_{2i} \sin\psi_i = Q_y$ [2b]

Equilibrium along Z axis: $\sum_{i=1}^{n} Q_{2i} \cos\alpha_{2i} \cos\psi_i = Q_z$ [2c]

Here n, is the number balls contacting the race and ψ is the azimuth angle, which is the angular position of the ball center relative to the inner race center. Note that since the inner race center is displaced relative to the fixed outer race center, this angle may be slightly different from the angular position of the ball center, θ, relative to the fixed outer race center.

Again, these three equations may be simultaneously solved to compute the three components, (X, Y, Z) , of the inner race displacement. Since the Hertzian load-displacement relation is nonlinear a solution to these equations is obtained iteratively. As shown schematically in figure 4, there are two iterative loops: for a prescribed inner race position the ball equilibrium equations are solved for each ball, then the race equilibrium is solved under the prescribed load on the bearing. The algebraic equations are nonlinear but the solution is straight-forward and it is rapidly obtained using the classical Newton-Raphson iterative techniques.

Since the solutions are obtained in terms of displacements, it is sometimes convenient to prescribe relative race displacement rather than applied load. This eliminates the iterations for the race and the load exerted corresponding to the input displacement is included in the output. AdoreQS, therefore, accepts either applied loads or displacements as inputs.

The formulation of cylindrical, spherical and tapered roller bearings is similar to the one presented above for a ball bearing.

Angular Velocities: In addition to computation of contact loads under the prescribed relative position of the bearing elements, the angular velocities of the balls or rollers must also be computed with prescribed race angular velocity. Also, since the centrifugal force in the equilibrium equations is determined by the orbital angular velocity of the rolling element, this computation has to be carried out at each computation of contact loads. In other words, the computation of angular velocities is within the rolling element equilibrium loop.

The ball angular velocity, ω, in an angular contact ball bearing is normally oriented at an angle β relative to the shaft axis, as shown earlier in the schematic presented in figure 3. Thus, the ball angular velocity is defined by its two components, along the *x* and *z* axes. The other unknown is the ball orbital velocity, $\dot{\theta}$, not shown in figure 3. For these three unknowns, three kinematic conditions on some type are required. The three conditions, commonly used in quasi-static models, are:

1. Pure rolling at center of the outer race contact
2. Pure rolling at center of the inner race contact
3. Some type of empirical kinematic constraint to define orientation of the ball angular velocity vector

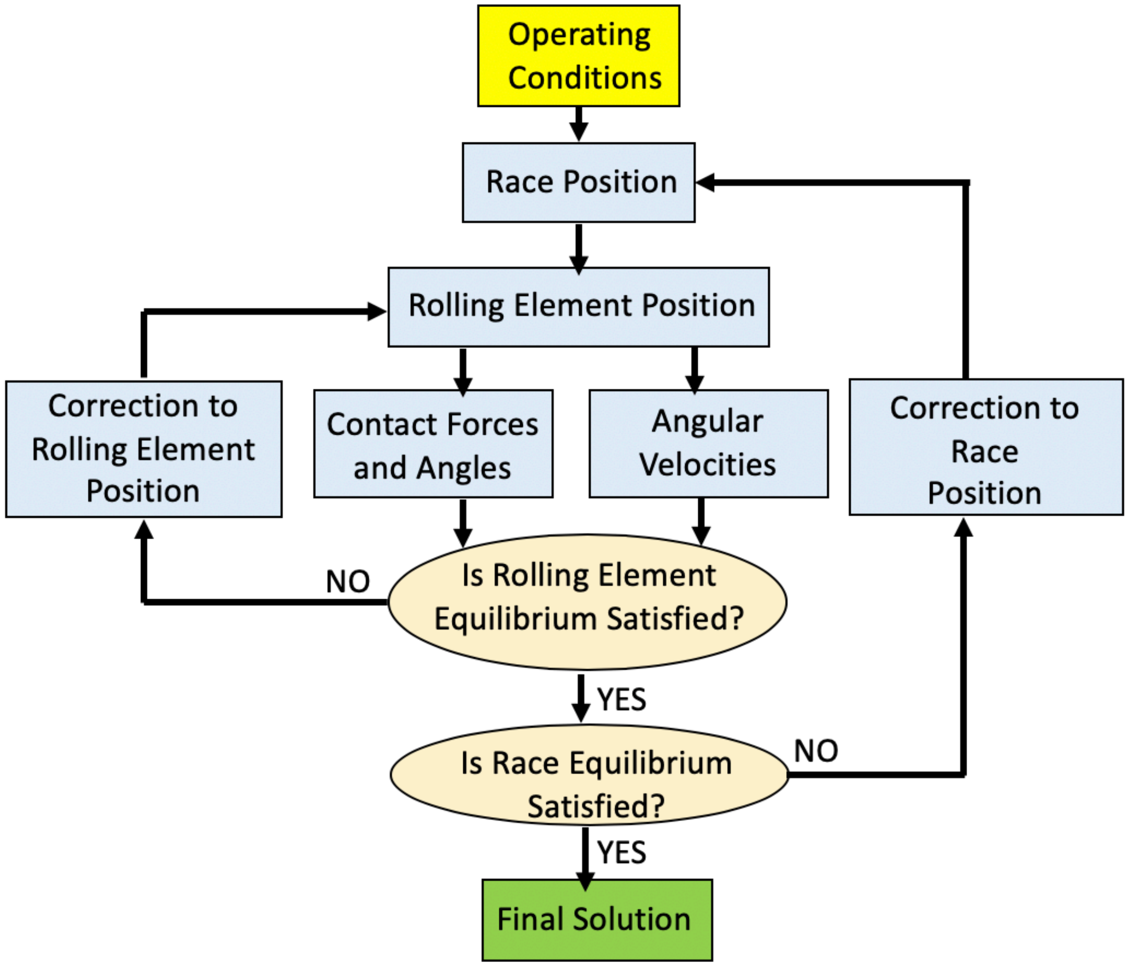

Figure 4. Schematic flow chart of the quasi-static equilibrium model.

For roller bearings, since the orientation of roller angular velocity is well defined along the roller axis, the first two kinematic conditions are adequate and the formulation is quite simple. For angular contact ball bearings, however, the required additional condition is prescribed in terms of an empirical constraint. AdoreQS provides an option to implement any one of the following three hypothetical constraints.

1. Race control hypothesis
2. Minimum energy hypothesis
3. Arbitrary orientation of the ball angular velocity vector

The following discussion of these constraints will assist the user in making an appropriate selection.

Race Control Hypothesis: Race control hypothesis was originally introduced by Jones (2). The hypothesis states that relative spin, component of ball angular velocity relative to the race about the load axis, normal to ball/race contact surface, shall exist only on the race which provides lesser of the spin moment. The race with no relative spin is called the controlling race. Assuming that the friction forces are proportional to the normal forces, Jones (2) has expressed the race control hypothesis strictly in terms of the Hertzian pressure distribution over an elliptical contact. Therefore, although this simple hypothesis provides the required equation for computation of ball angular velocities, it does not implement any tribological behavior in the ball/race contact. In other words, the lubricant traction model provides no input to the analysis.

A typical ball/race slip distribution with the race control hypothesis is shown in figure 5, which plots a solution with outer race control, when the ball spin is restricted to the inner race.

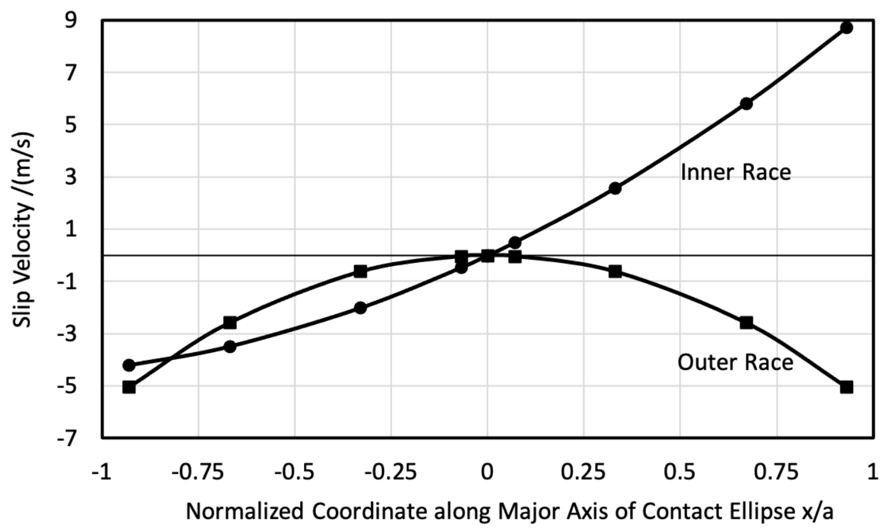

Figure 5. Typical ball/race slip distribution in the contact under outer race control hypothesis.

Note that this solution conforms to the three conditions listed above, all of which are, of course, empirical. Pure rolling points (no relative slip) are arbitrarily assumed to be at center of the contact on both outer and inner races (conditions 1 and 2) and in the solution shown in figure 5, the ball spins only on the inner race (condition 3, outer race control). The frictional dissipation in the contact may certainly be calculated with any prescribed traction-slip relation or lubrication model, but the lubricant model has no input in generating the solution.

Minimum Energy Hypothesis: The validity of race control hypothesis has often been questionable, particularly under well lubricated conditions, where the ball angular velocity may have a spin component on both races. After noting such a behavior in a well lubricated precision ball bearing, Gupta (3) proposed an alternate minimum energy hypothesis, where the ball angular velocity vector orientation is computed by minimizing the frictional energy dissipated in the ball-to-race contacts under prescribed lubrication condition. This requires a close integration of the lubricant traction model with bearing kinematics, which is indeed a compute intensive task. Therefore, the model was initially implemented in bearing dynamics code, ADORE, only as a simplified modification of the race control solution with a prescribed lubricant traction model. Recently, with the advent of modern high-speed computing tools, Gupta (4) has carried out a more detailed analysis of the problem and the model is now implemented as an independent alternate kinematic hypothesis. This implementation is available in AdoreQS as an alternate kinematic hypothesis.

A schematic review of the ball/race contact is shown in figure 6. It is noted that since the contact is actually along a curved surface, there may be either one or two points of pure rolling (no relative slip) along the major axis of the contact ellipse. Thus, in addition to the orientation of the ball angular velocity vector, the points of pure rolling are additional independent variables, which define the frictional dissipation in the contact. The total frictional dissipation or contact energy must, therefore, be minimized as a function three independent variables, orientation of ball angular velocity vector, points of pure rolling on the outer and inner races. Such an optimization is accomplished in a numerically systematic fashion while implementing the minimum energy hypothesis in AdoreQS.

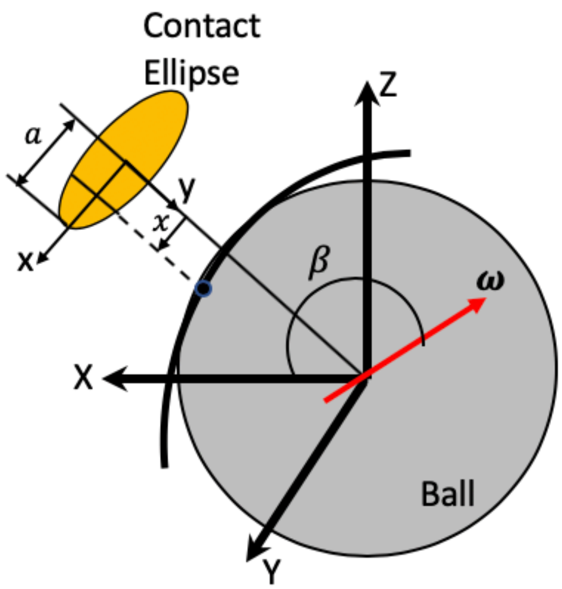

Figure 6. Schematic of the ball/race contact.

While the detailed procedure has been documented by Gupta (4), the implementation is outlined in the greatly simplified flow chart presented in Figure 7. First the orientation of ball angular velocity vector, β, is assumed, then using the applicable kinematic equations, contact load solutions and the prescribed lubricant traction model the total frictional dissipation in the outer and inner race contacts, or the total energy dissipated in the contacts is minimized as a function of pure rolling points in the outer and inner race contacts. This leads to the minimum energy dissipated in the contacts, q_m, as a function of the prescribed ball angular velocity vector orientation, β. This dissipation is then minimized as a function of β in an iterative fashion.

A typical ball/race contact slip distribution obtained with the minimum energy hypothesis is shown in figure 8. This may be compared with the corresponding solution, obtained with outer race control hypothesis, shown earlier in figure 5. It may be noted that the outer race now has two points of pure rolling and they are not symmetric with respect to center of the contact. Also, the point of pure rolling on the inner race is no longer in the center of contact. This implies that both the outer and inner race contacts have both a spin and roll component.

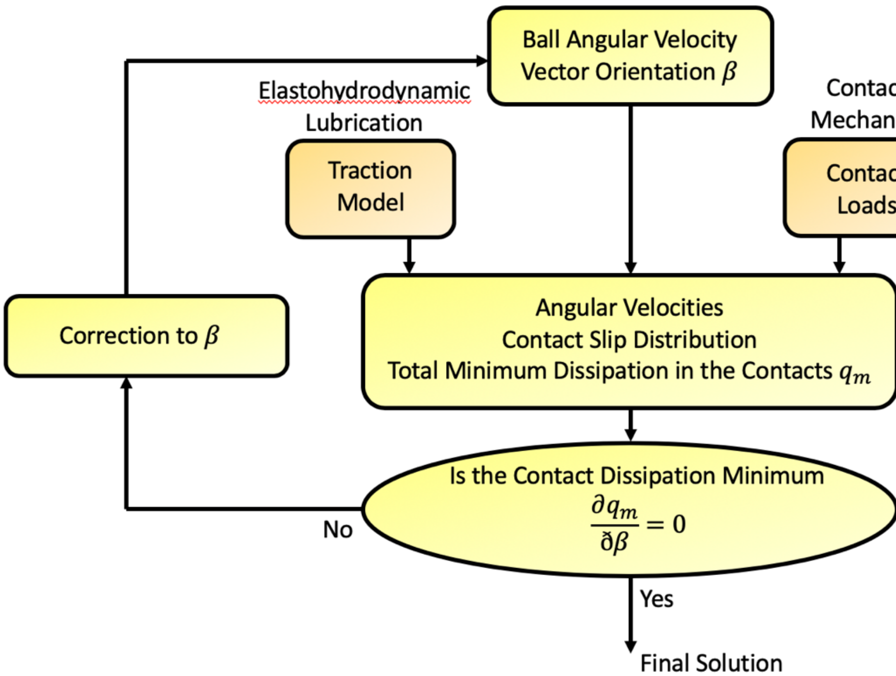

Figure 7. Implementation of minimum energy hypothesis in AdoreQS.

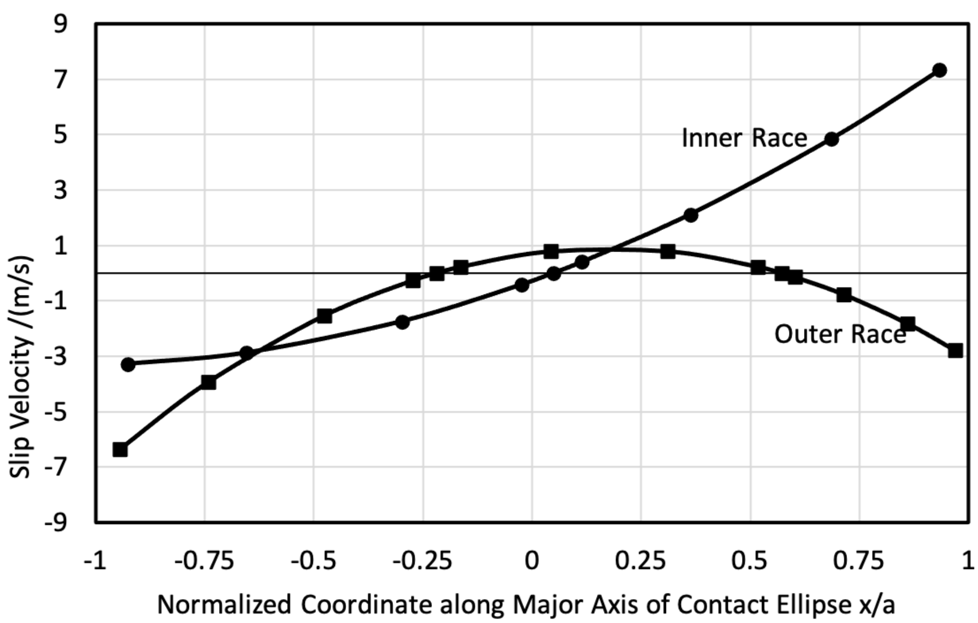

Figure 8. Typical contact slip distribution in ball race contact with the minimum energy hypothesis.

Corresponding to the contact slip distribution solutions presented figure 8, the minimum energy dissipated as a function of the orientation of ball angular velocity, β, is plotted in figure 9. The minimum point on this solution represents the final solution for the orientation of the ball angular velocity vector.

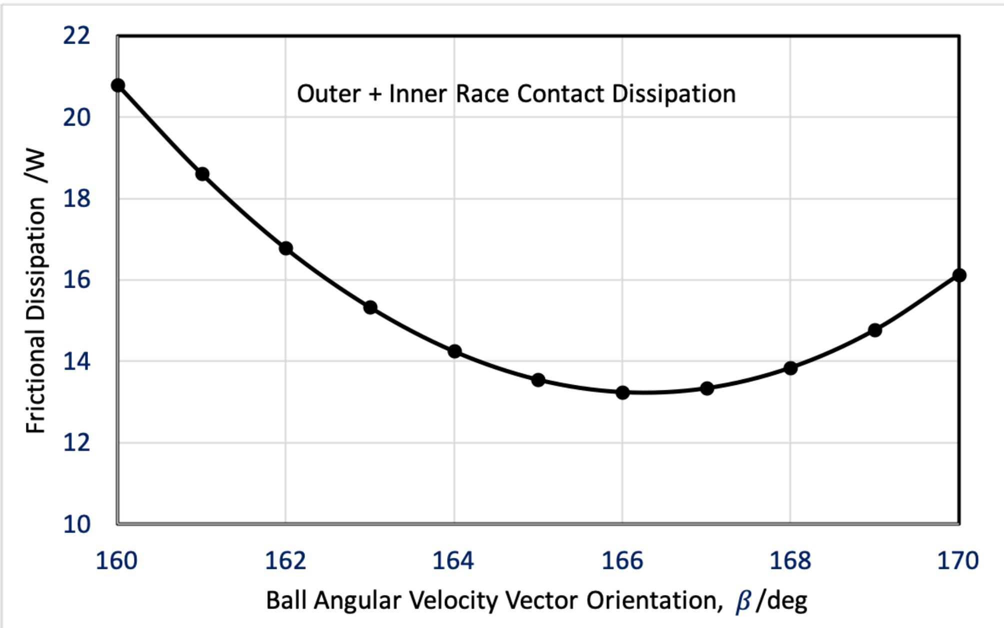

Figure 9. Typical behavior of frictional dissipation in ball/race contacts as a function of the orientation of ball angular velocity vector.

Unlike the simple race control hypothesis, which has no interaction with the lubricant traction model, the minimum energy hypothesis results in three iterative loops for computing the ball angular velocity within the ball equilibrium loop, and the lubrication traction model is called within the inner most loop. Thus, depending on complexity of the lubricant traction model the process may be quite compute intensive. After obtaining a solution, AdoreQS saves it for applying it as initial guess for the next set of iterations. This reduces the number of iterations and saves substantial computing effort, but the overall process is still significantly more compute intensive in comparison to that used in implementation of the simple race control hypothesis. In a ball bearing with combined axial and radial load, since solution of each ball may be different from the other the computation becomes more intensive. The user must be aware of these computational requirements when using the minimum energy hypothesis.

In a parametric evaluation of bearing performance, Gupta (4) has demonstrated that the true dynamic steady-state contact heat generation solution obtained with arbitrary initial conditions closely converges to the one determined by the minimum energy hypothesis. Thus, AdoreQS may be a viable design tool to parametric evaluate reduced contact heat generation and increased contact stress when evaluating hybrid against all steel bearings. Once a preliminary design is developed with such a parametric study, the more rigorous dynamic analysis may be undertaken to validate the design and carryout final optimization if necessary. Hopefully, AdoreQS, with the introduction of the minimum energy hypothesis, will serve such a purpose.

Prescribed Ball Angular Velocity Vector Orientation: The final option for a kinematic constraint for computation of ball angular velocity in an angular contact ball bearing is arbitrary specification of the orientation of the angular velocity vector. Since this computation is already carried out under the minimum energy hypothesis, it is offered as an additional option in AdoreQS. It should, however, be noted that when the bearing has a combined thrust and radial load the ball angular velocity varies from ball to ball and so does the orientation of the angular velocity vector. Therefore, the use of this option may only be realistic in thrust loaded angular contact ball bearing.

Life Modeling

AdoreQS offers fairly comprehensive modeling of rolling bearing fatigue life. During the recent years there have been significant advancements in both the basic life models and models for modification of the basic life to allow for life enhancement resulting from the modern materials manufacturing and processing techniques, operating conditions and lubrication effects. AdoreQS incorporates all these advancements and provides life predictions with the various life models. The user, therefore, has the capability to compare life predictions with the different models and make an informed decision for the application under considerations.

Original Lundberg-Palmgren (LP) Model: The most commonly used fatigue life model is due to Lundberg and Palmgren (5,6). The model is developed on the basis of unpublished experimental fatigue life data on bearings made with pre-1950 AISI 52100 bearing steel. Analytically, since fatigue failure, like many other failures, is a statistical process, the Lundberg-Palmgren model is based on the classical Weibull hypothesis (7,8) which states the survival probability to be proportional to the product of a stress function and the stressed volume. After a review of failed bearings, Lundberg and Palmgren noticed that the fatigue spall on the bearing race is initiated below the surface and it is often close to the edge of the contact. They, therefore, postulated the stress function as a cyclic maximum subsurface orthogonal shear stress, which occurs close to the edge of contact, raised to a certain empirical exponent. In addition, since the failure is initiated below the surface, they added inverse proportionality to the depth, at which the maximum orthogonal shear stress occurs, raised to another empirical exponent. The fundamental Lundberg-Palmgren life equation is, therefore, written as:

$$\left(\frac{1}{L_{10}}\right)^m = K_{LP}\frac{\tau_o^c V_o}{z_o^h} \qquad [3]$$

where L_{10} is the basic life with a 10% failure or 90% survival probability, τ_o is the maximum orthogonal subsurface shear stress, z_o is the depth below the surface at which the stress occurs, V_o is the applicable stressed volume, and K_{LP}, c and h are empirical constants derived by fitting the model predictions to experimental bearing life data. Also, m, the Weibull slope is derived from dispersion of the experimental life data. Note that while using equation [3] to derive life, the shear stress exponent is c/m. In other words, data variability is included in the stress exponent.

Using the Hertz contact theory, Lundberg and Palmgren related the maximum orthogonal shear stress and its depth to the applied contact load and bearing geometry. Also, for simplicity the entire material volume above the maximum orthogonal shear stress around the rolling track on the race is assumed to be stressed. In addition, the material properties are absorbed in the proportionality constant. This permits the life equation to be expressed only as a function of contact load and the applicable geometrical parameters of the bearing, as documented by Harris and Kotzalas (9). This leads to the introduction of "dynamic load capacity", which represents a contact load under which the bearing race will survive one million revolutions. The model consists of life equations for the outer and inner races only. However, since the life predictions are correlated to bearing life, life of rolling elements is inherently included in the model. The simplicity of the life equations makes the model very easily usable and it is, therefore, most widely used and considered as state-of-the-art in rolling bearing design.

Although the fundamental life equation [3] is applicable to both point and line contacts, Lundberg and Palmgren have presented independent empirical life constants for ball and roller bearings (9). Perhaps, by independent correlation of model predictions to experimental ball and roller bearing life data. This makes the life equations for ball and roller bearings independent, although they both share the same fundamental hypothesis.

Since the life equations only contain geometrical parameters of the bearings and the applied contact load, and all material properties are part of the empirical proportionality constant, the life equations are free of any material property input. Although a change of material properties alters the contact load and geometry, which is input to the life model, the base life constant, which is based on pre-1950 AISI 52100 bearing steel, remains unchanged. The model, therefore, has some limitations in life predictions when the material properties are altered, either due to change of operational temperature or change of materials as in modern hybrid bearings, where the rolling elements are made from a ceramic, such as silicon nitride.

Generalized Lundberg-Palmgren (LP) Model: With due recognition of the above materials limitation, Gupta and Zaretsky (10) have generalized the Lundberg-Palmgren formulation, and they have presented a new generalized form, in terms of distinct geometrical and materials parameters. In addition, this generalized model expresses life in terms of contact stress rather than load. This led to the introduction of a "dynamic stress capacity", which represents a ball/race contact stress under which the raceway will survive one million revolutions. In addition, the validity of the fundamental equation [3] and the basic proportionality constant is maintained for both ball and roller bearings. Thus, the ball and roller bearing life constants are related to each other. Therefore, model validity for ball bearings, also validates the model for roller bearings. Life predictions with the generalized models have been validated against those obtained with the original model for bearings made with AISI 52100 bearing steel and the results are shown to be closely identical (10).

In addition to the introduction of distinct materials parameter, where the materials properties may be input into the life model, the generalized model segments bearing life into life of the two races and rolling elements. This permits independent variation of empirical life exponents and constants for the races and rolling elements. Hence, life of hybrid bearings may be more precisely modeled.

Gupta-Zaretsky (GZ) Model: In addition to generalizing the Lundberg-Palmgren model, Gupta and Zaretsky (10) presented a new life model based on early work of Zaretsky (11). This model makes three fundamental modification to the Lundberg-Palmgren model:

- Data variability in shear stress exponent in the fundamental life equation is eliminated.
- By assuming rolling contact fatigue as a high-cycle fatigue process, variation of life with depth of the critical failure stress is eliminated.
- The critical failure stress is assumed to be the maximum shear stress, rather than the maximum orthogonal shear stress as in the Lundberg-Palmgren model.

The fundamental life equation for this newly formulated model, corresponding to equation [3] of the Lundberg-Palmgren formulation is written as:

$$\left(\frac{1}{L_{10}}\right)^{m} = K_{GZ}\tau_m^c V_m \qquad [4]$$

where τ_m is the maximum subsurface shear stress, V_m is the applicable stressed volume, and K_{GZ} and c are empirical constants derived by fitting the model predictions to experimental bearing life data. Note that equation [4] more closely conforms to the original Weibull hypothesis, which relates survival probability to the product of a stress function and the stressed volume.

The resulting life model provides a somewhat higher stress-life exponent, which contributes to significantly higher bearing lives at very light loads in comparison to the Lundberg-Palmgren predictions.

More recently, based on the only available life data on silicon nitride balls obtained by Parker and Zaretsky (12), Gupta and Zaretsky (13) have developed new life constants and exponents for modeling life of silicon nitride balls. These constants and exponents have been incorporated in both the generalized Lundberg-Palmgren and the Gupta-Zaretsky models. Since the ball and roller bearing constants are related in these generalized models, modeling life of hybrid roller bearings also becomes possible.

Ioannides-Harris (IH) Model: Ioannides and Harris (14) have proposed that rolling bearing life is infinite when the critical failure stress is below a limiting stress, and when the limiting stress is zero the model converges to the Lundberg-Palmgren equation. Although, Ioannides and Harris (14) state the model applicability with any critical failure stress, they used the maximum orthogonal shear stress as the failure stress, just to conform to the standard Lundberg-Palmgren model when the limiting stress vanishes. The model is implemented as failure stress modification in the Lundberg-Palmgren model, where the applicable failure stress is reduced by the limiting stress, and life is infinite at stress below the limiting stress.

More recently, Gupta (15) investigated failure stress modification in a more generalized fashion. First, the failure stress in the Gupta-Zaretsky model is modified by applicable compressive residual stress to model the role of residual stress in life modeling. The results are very well validated against experimental bearing life data available with case hardened race materials, containing significant residual stresses as a result of the manufacturing process. Tensile hoop stresses are also incorporated as a part of this failure stress modification to investigate the impact of these stresses on life of high-speed bearings. Second, the generalized stress modification approach is applied to the Ioannides-Harris model using an octahedral shear stress, as the failure stress. A limiting value of this stress is related to the von-Mises stress. A new version of the Lundberg-Palmgren model is also developed by replacing the failure stress from maximum orthogonal shear stress to maximum octahedral shear stress. Thus, when the limiting stress is reduced to zero, life predictions conform to the Lundberg-Palmgren predictions. For convenience a limiting stress factor is introduced in the formulation, such that when this factor is 1, the model conforms to the Ioannides-Harris model.

With the implementation of octahedral shear stress, incorporation of residual and hoop stresses becomes straight forward. Thus, the role these stresses along with a possible limiting stress may be readily evaluated. Also, when the limiting stress is set to zero, the role of residual and hoop stresses may be modeled with Lundberg-Palmgren type formulation.

With the introduction of limiting stress factor it is also possible to make life predictions conforming to the ISO 281 standard (16), which states bearing life to be infinite when the contact stress is below 1.50 GPa with AISI 52100 bearing steel. This condition is simulated by setting the limiting stress factor to 1.28, as demonstrated by Gupta (15).

While there is substantial controversy on existence of a limiting stress in rolling contacts (17,18), the generalized implementation of the failure stress modification approach permits the following three predictions:

- Ioannides-Harris predictions when limiting stress factor is 1.0
- Lundberg-Palmgren type predictions, with maximum octahedral shear as the failure stress, when limiting stress factor is 0.
- ISO 281 prediction when limiting stress factor is 1.28.

The model also incorporates residual stress so the effect of these stresses combined with the limiting stress may be investigated. When the limiting stress factor is set to zero, the model permits modeling the role of residual and hoop stress with the Lundberg-Palmgren model, which cannot be done with the standard formulation using the orthogonal shear stress.

Models for Life Modification: The above models provide fatigue life estimate strictly based on subsurface fatigue under a defined failure criterion. The estimated life is, perhaps, most conservative. In practice the observed life is significantly enhanced due to several materials and operational factors. For example, modern material manufacturing and processing techniques result in significant reduction in sites where subsurface fatigue cracks may originate. Likewise, full film lubrication greatly enhances surface interaction and results in improved life. In order to allow for these enhancements, life modification factors are applied on the computed basic life to estimate a more realistic life of a rolling bearing under prescribed operating environment. The STLE recommended life modification factors (19) are the easiest to apply and they are most widely used in the industry. These factors are simply applied on the computed basic life of the bearing as multipliers. The factors are supported by substantial experimental data and considered fairly realistic for a wide range of practical applications. Tallian (20) has developed more rigorous life modification algorithms, which modify subsurface fatigue life at the level on individual contacts in the bearing. The resulting enhanced lives are again validated against experimental data. However, since these factors are applied at the individual contact level, the application is computationally more complex. For comparison purpose, AdoreQS provides life modification factors with both the STLE and Tallian models.

Technical Questions and Comments

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