Generalized Models for Rolling-Element Fatigue
Part 1: Analytical Formulation

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Generalized Models for Rolling-Element Fatigue

Part 1: Analytical Formulation

- Introduction
- Fundamental life equation
- Lundberg-Palmgren (LP) equation
- New life equation
  - Modification of the LP equation
- Computation of model constants from Regression Analysis of Experimental Data
- Summary

Generalized Models for Rolling-Element Fatigue

Part 1: Analytical Formulation

Introduction

- Lundberg-Palmgren formulation of 1940-50’s
  - 52100 Bearing Steel of 1940’s
  - Constant Material Properties
  - Current state of the art
- Modern materials – Processes – Ceramics
  - Property variation with temperature
- Currently used original LP models inadequate
- Models based on limiting shear stress (Ioannides & Harris 1985)
  - Controversy on existence of shear limit

Generalized Models for Rolling-Element Fatigue

Part 1: Analytical Formulation

Present Work Scope

- New Generalized LP model
  - Variable material properties
- Zaretsky Model (1987)
  - Shear exponent independent of life scatter
  - Life independent of depth of failure stress
  - Failure stress is maximum subsurface shear stress compared to maximum orthogonal shear stress in LP model

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Fundamental Life Equation

Lundberg-Palmgren-Weibull Equation

- Applicable to both point and line contacts

$$\ln\frac{1}{S} \sim \tau_o^c V z_o^{-h} N^m \qquad \ln\frac{1}{S} = K_{LP}\tau_o^c V z_o^{-h} N^m$$

$\tau_o \sim p_H$ (Max Hertz Contact Stress)

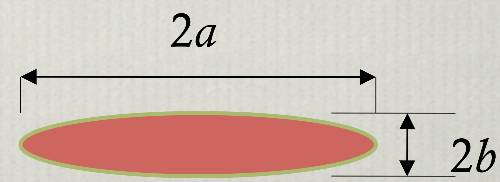
$V \sim a$ (Major Half Width) z_o (Shear Stress Depth) d (Track Dia)

$z_o \sim b$ (Minor Half Width)

$N =$ Life in contact cycles

$S =$ Survival probability

c, h, m are empirical exponents



Implementation for Bearing Life Computations

$$\ln\frac{1}{S} = K_{LP}\tau_o^c V z_o^{-h} N^m$$

- Using elastic contact solutions express shear stress, and contact half width in terms of applied load
  ➔ Thereby obtain a load-life relations

- From the above compute a load, Q_c, under which the bearing will survive for one million revolutions with a given survival probability
  ➔ This is defined as dynamic load capacity

- Now define life as $$L = \left(\frac{Q_c}{Q}\right)^p$$

$$L = \text{ Life, } Q = \text{ Applied Load, and } p = \text{ Load - Life exponent}$$

Elasticity Solution
Contact Pressure

- Point contact

$$p_H = \frac{3}{2}\frac{Q}{\pi ab}$$

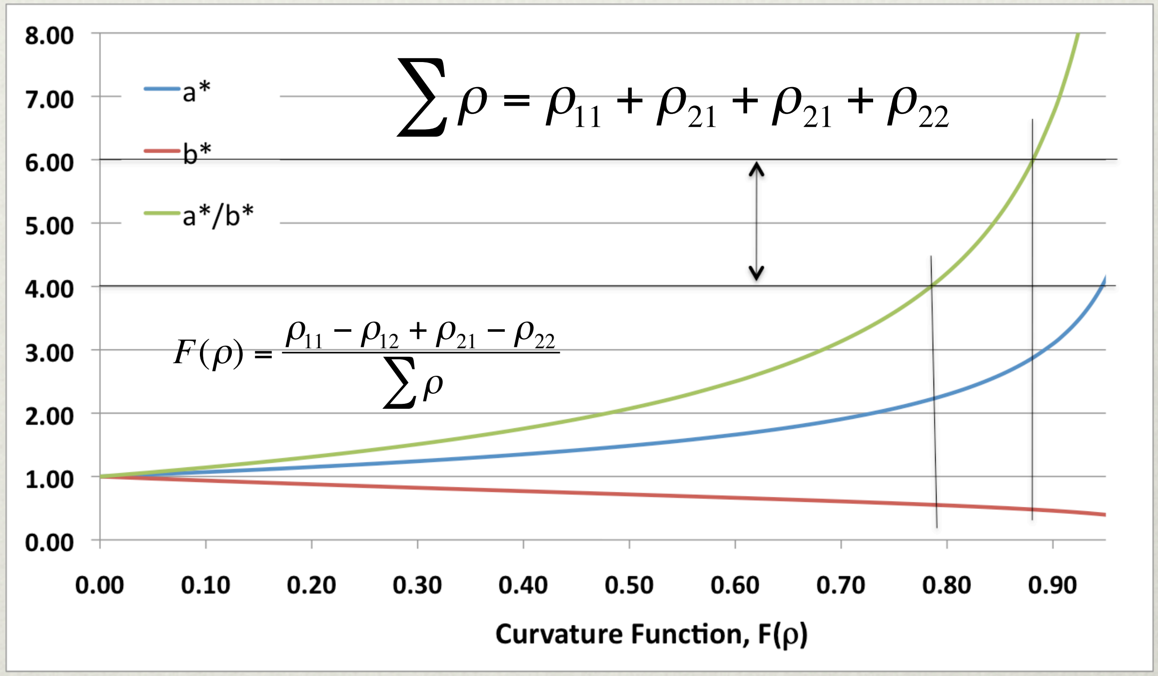
- Line contact

$$p_H = \frac{Q}{\pi ab}$$

Elasticity Solution

Contact Half Widths

- ❖ Point contact



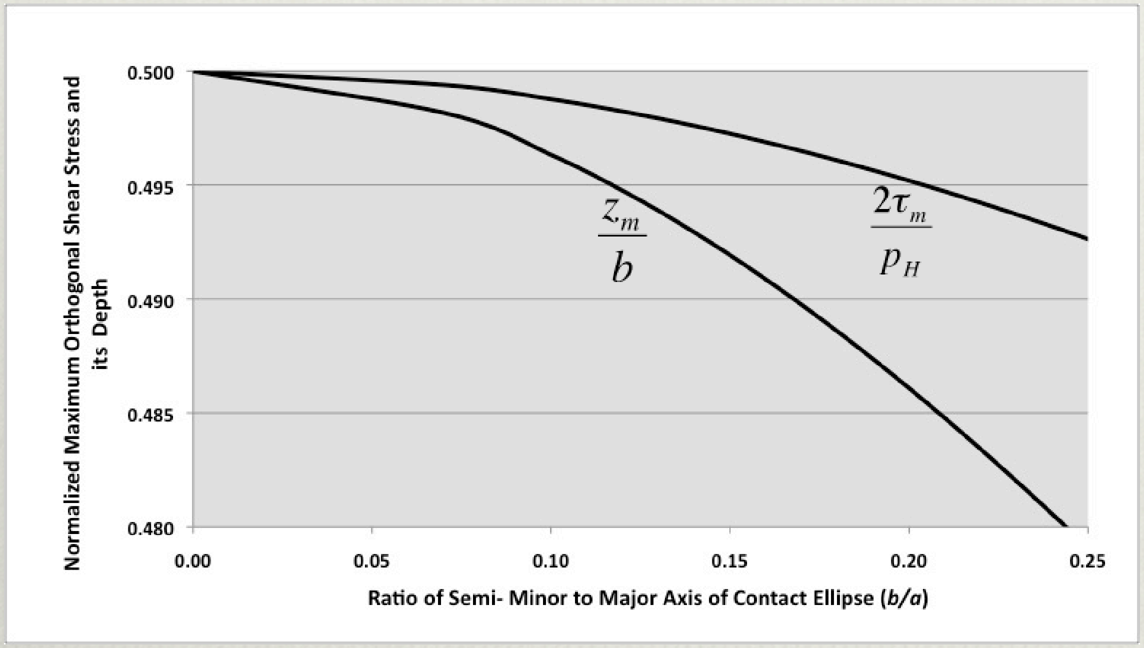
- ❖ Line contact

a = constant (roller half length)

$$b = \left[\frac{2}{\pi a \sum \rho} \frac{Q}{E'} \right]^{1/2}$$

Elasticity Solution

Subsurface Shear Stress and Depth



Line Contact

$$b/a = 0$$

Point Contact

$$0.20 \le b/a \le 0.25$$

For both Line & Point Contact

$$\frac{2\tau_o}{p_H} = \frac{1}{2} \quad \text{or} \quad \frac{\tau_o}{p_H} = \zeta = \frac{1}{4} \qquad \frac{z_o}{b} = \xi = \frac{1}{2}$$

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Original Lundberg-Palmgren (LP) Life Equation (1947-52)

Empirical Exponents and Model Constants

- Empirical model constants
  - Point contact $A = 2.4640 \text{ x}10^{7} \;\; \text{N/m}^{1.80}$
  - Line contact $B = 1.9817 \text{ x}10^{8} \; \text{N/m}^{1.852}$
  - Survival probability $S = 0.90$
- Empirical exponents $c = \dfrac{31}{3} \qquad h = \dfrac{7}{3} \qquad m = \dfrac{10}{9}$
- Life Expressed in million of race race revolution with contact frequency defined as

$$u = \left| \frac{\text{Orbital Ang Velo - Race Ang Velo}}{\text{Race Ang Velo}} \right|$$

Original LP Equations (1947-1952)

Based on work by Weibull

- Point Contact – Ball Bearings

$$Q_c = A\left[\left(\frac{2f}{2f-1}\right)^{0.41}\left(\frac{1\mp\dfrac{D\cos\alpha}{d_m}}{1\pm\dfrac{D\cos\alpha}{d_m}}\right)^{1.39}\left(\frac{D}{d_m}\right)^{0.30} D^{1.80}\right]\left[u^{-1/3}\right] \qquad L=\left(\frac{Q_c}{Q}\right)^3$$

- Line Contact – Roller Bearings

$$Q_c = B\left[\left(1\mp\frac{D}{d_m}\right)^{29/27}\left(\frac{D}{d_m}\right)^{2/9} D^{29/27}\left(l=2a\right)^{7/9}\right]\left[u^{-1/4}\right] \qquad L=\left(\frac{Q_c}{Q}\right)^4$$

- Limitation – elastic properties are constant and part of model constant

More Generalized Life Equation

- Life equation is a function of bearing geometry, material properties and operating conditions:

$$L = K\, f(E,\upsilon)\, f(\text{Geometry})\, f(Q,\Omega)$$

- For Unit Life (one million rev) a characteristic load, *Dynamic Capacity*, is defined as:

$$Q_c = K\, f(E,\upsilon)\, f(\text{Geometry})\, f(\Omega)$$

- The life is then defined as:

$$L = (Q_c / Q)^n$$

- Constant K in the equation for Q_c is determined by correlating predicted life with experimental data

New Generalized LP Life Equation

Distinct Geometrical and Materials Factors

- Point contact

$$Q_{cLP} = \Phi_{LP} A_{LP} \kappa_{LP}^{-0.30} \left(\frac{G_{LP}}{\lambda_E} \right)^{2.10} u^{-1/3} \qquad L_{10} = \left(\frac{Q_{cLP}}{Q} \right)^{3}$$

- Line contact

$$\overline{Q}_{cLP} = \overline{\Phi}_{LP} B_{LP} \kappa_{LP}^{-0.2222} \left(\frac{\overline{G}_{LP}}{\lambda_E} \right)^{1.2963} u^{-0.2469} \qquad L_{10} = \left(\frac{\overline{Q}_{cLP}}{Q} \right)^{4.050}$$

- Newly introduced materials property factor

$$\lambda_E = \frac{E'}{E'_{52100}}$$

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Generalized Life Equations
Zaretsky Model (1987)

- Recall fundamental Lundberg-Palmgren equation

$$\ln\frac{1}{S} \sim \tau_o^c V z_o^{-h} N^m \quad \text{or Life} \quad N \sim \frac{\left(\ln\frac{1}{S}\right)^{1/m}}{\tau_o^{c/m} V^{1/m} z_o^{-h/m}}$$

- Zaretsky proposes
  - Shear stress exponent should be independent on data variability
  - Life should not depend explicitly on depth of critical shear stress
  - Critical shear stress should be maximum shear stress not maximum orthogonal shear stress

Generalized Life Equations

GZ Equation – Modified case of LP Equation

- Shear stress exponent should not be dependent on data variability

$$\text{Shear Exponent}: \quad c \rightarrow cm$$

- Life should not depend on depth of critical shear stress

$$\text{Depth Exponent}: \quad h \rightarrow 0$$

- Critical shear stress should be maximum shear stress not maximum orthogonal shear stress

$$\zeta = \frac{\tau_m}{p_H} \rightarrow 0.30 \quad \& \quad \xi = \frac{z_m}{b} \rightarrow 0.786$$

Generalized Life Equations
Zaretsky Model Implementation

- Zaretsky Model, although proposed in 1987, it has never been implemented as a viable rolling bearing life equation
- Thus the Zaretsky Model is the basis for the newly proposed **Gupta-Zaretsky (GZ)** life equation

New Generalized GZ Life Equation
Distinct Geometrical and Materials Factors

- Point contact

$$Q_{cGZ} = \Phi_{GZ} A_{GZ} \kappa_{GZ}^{-0.2225} \left(\frac{G_{GZ}}{\lambda_E} \right)^{1.5549} u^{-0.2472} \qquad L_{10} = \left(\frac{Q_{cGZ}}{Q} \right)^{4.0444}$$

- Line contact

$$\overline{Q}_{cGZ} = \overline{\Phi}_{GZ} B_{GZ} \kappa_{GZ}^{-0.1602} \left(\frac{\overline{G}_{GZ}}{\lambda_E} \right)^{0.8398} u^{-0.1780} \qquad L_{10} = \left(\frac{\overline{Q}_{cGZ}}{Q} \right)^{5.6167}$$

- Newly introduced materials property factor

$$\lambda_E = \frac{E'}{E'_{52100}}$$

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Computations of Model Constants

- Lundberg-Palmgren data is unpublished, besides the model is based on Pre-1940 materials

- Based on recent work by Gupta, Oswald and Zaretsky (*STLE 2013 Annual Meeting*) NASA/GRC data on 120 mm Turbine Engine Bearing used to compute model constants A_{LP} and A_{GZ}

- B_{LP} and B_{GZ} derived from validated point contact constants A_{LP} and A_{GZ}

Computations of Model Constants

- Computation of model constants by regression of experimental data
- 126 Ball bearing life tests in four data sets with failure indices of 10/27, 14/27, 11/26 and 6/26
- Computed values of point contact constants

$$\Phi_{LP} = 1.0 \qquad A_{LP} = 1.2197 \text{ x}10^{7} \text{ N/m}^{1.80}$$

$$\Phi_{GZ} = 1.0 \qquad A_{GZ} = 8.3368 \text{ x}10^{5} \text{ N/m}^{1.332}$$

- Line contact constants derived from relation between point and line contact constants

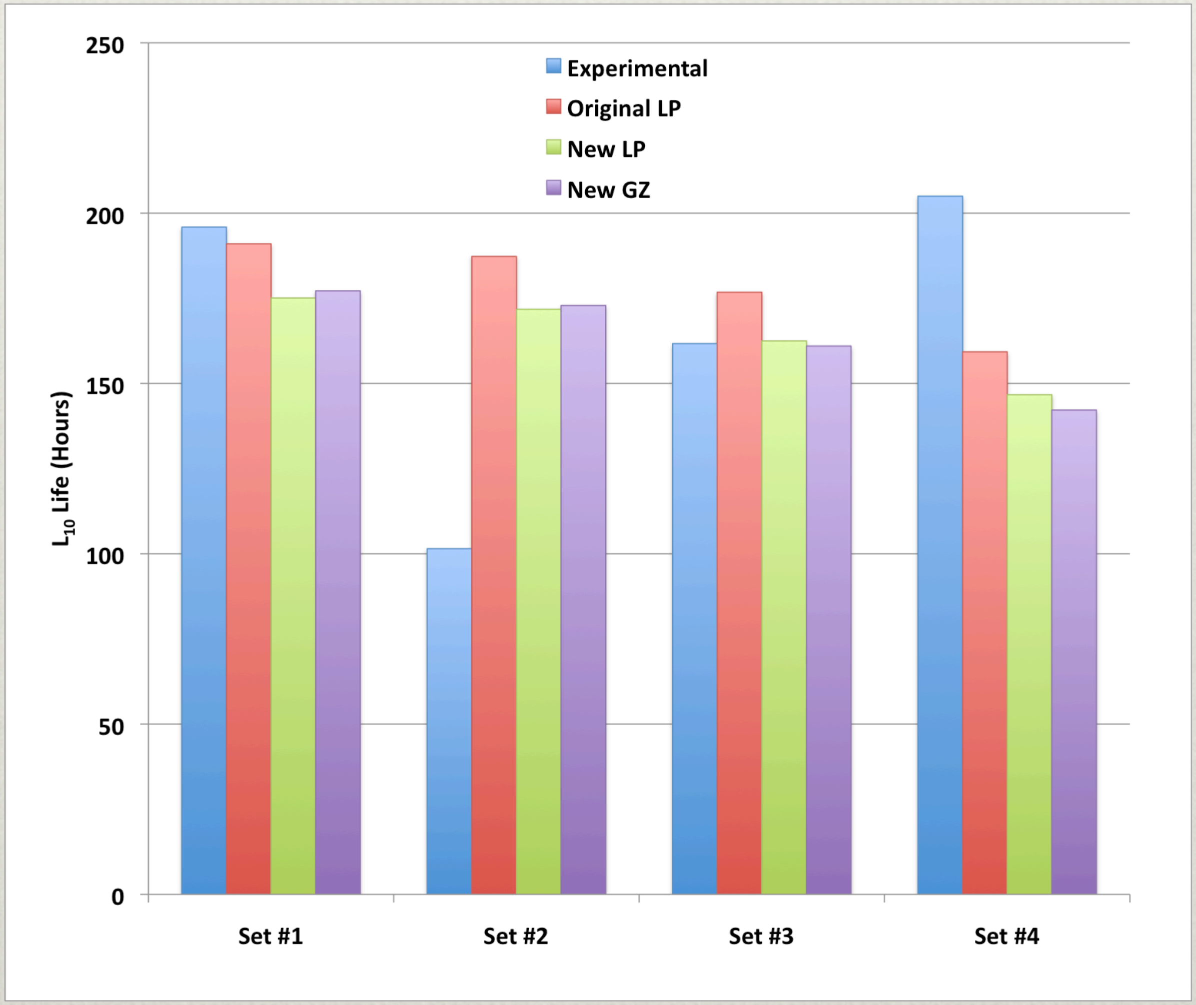
$$\overline{\Phi}_{LP} = 0.90 \text{ (For Calibration)} \qquad B_{LP} = 9.7020 \text{ x}10^{7} \text{ N/m}^{1.852}$$

$$\overline{\Phi}_{GZ} = 0.90 \qquad B_{GZ} = 2.0670 \text{ x}10^{7} \text{ N/m}^{1.519}$$

- The adjustment factor of 0.90 was necessary to match the new LP life with the original LP life in absence of any new experimental data

Computations of Model Constants

Comparison of fitted Models to Experimental Data

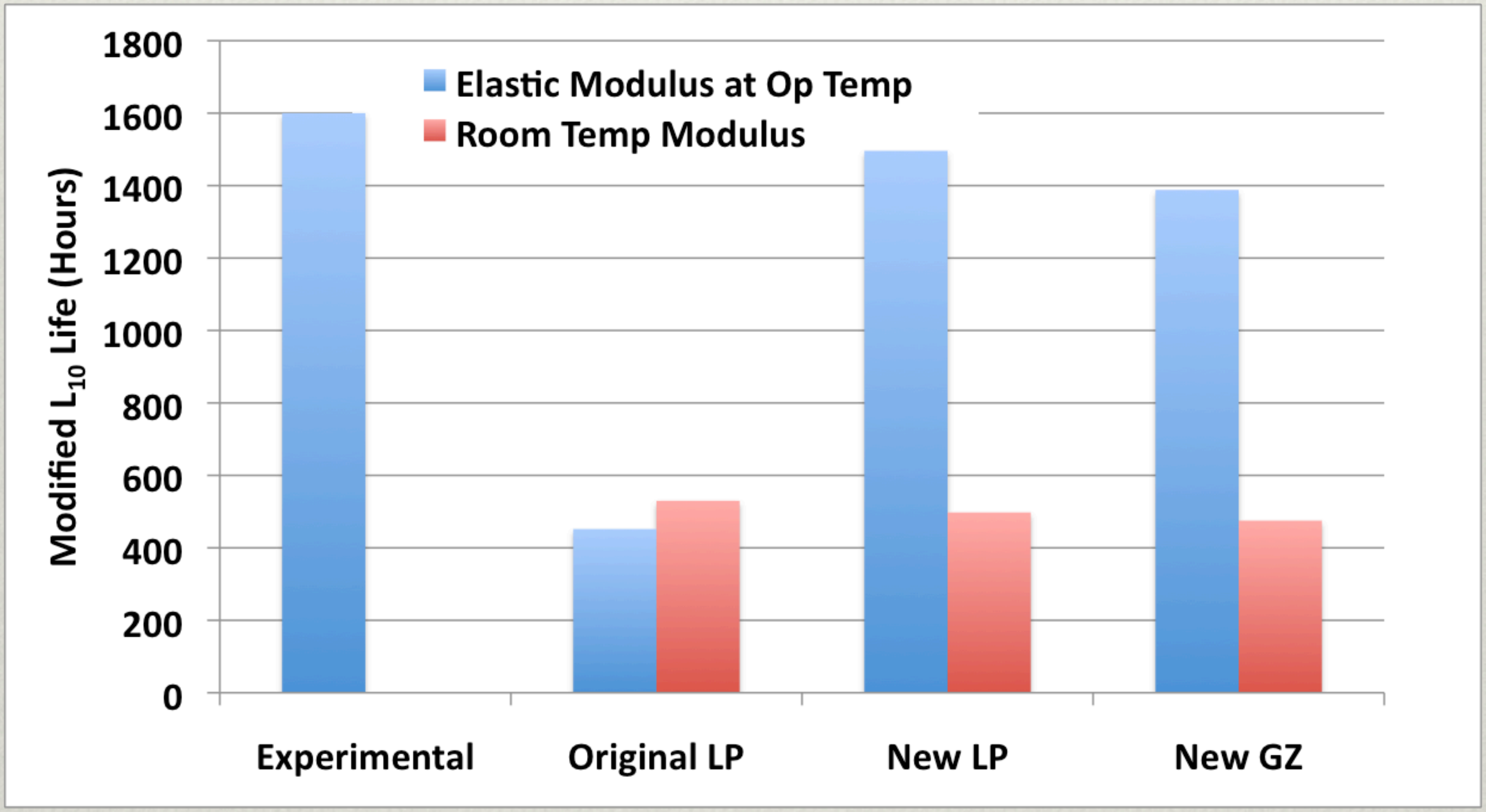


Validation of Model Predictions

- Using the estimated model coefficients predict life for another bearing for which experimental life data is available
- 3 MDN AISI M-50 VIM-VAR bearing lubricated with MIL-L-23699 operating at 492^{o}K
  - Experimental data with failure index of 6/30
- Elastic properties of M50 at 492^{o}K
- Generalized models permit elastic modulus variation

Validation of Model Predictions

Model Predictions versus Experimental Data



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Summary

- Generalized models development with distinct materials and geometrical parameters
- Unlike original LP model new generalized models permit arbitrary variation in elastic properties
- In addition to updating LP life equation new GZ life equation is introduced
- Model predictions validated against experimental data
- Generalized models provide relationship between point and line contact model constant

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