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Current Status of and Future Innovations in Rolling Bearing Modeling

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Current state-of-the-art in modeling the performance of rolling bearings is reviewed in terms of fundamental analytical formulations and the development of computer codes for performance simulations. Some of the basic equations, which constitute the foundation of the various types of models, are reviewed before presenting a schematic approach for the development of rolling bearing models. Some of the key developments over the last several decades that have led to the current status of rolling bearing modeling are presented. Though some of the models are restricted to the developing organizations, and their use is only available in terms of application support, others have been packaged in the form of commercially available software products. These models provide immediate practical implementation of several tribological disciplines in their most up-to-date and advanced form. With the advancements in high-speed computing technologies, solutions to the most sophisticated analytical formulations have become possible. However, the parallel advancement in rotating machinery systems has continued to challenge the state-of-the-art of rolling bearing modeling and in order to meet the future requirements, further developments in certain areas are required. Such requirements include improvements in lubricant behavior, development of lubricant and material property databases, more advanced thermal management and modeling of bearing interactions, more sophisticated models to estimate energy dissipated in lubricant churning and drag, and implementation of modern object-oriented computing languages for better support of modeling software products on the current and anticipated future computer systems.

KEY WORDS

Rolling Bearing; Modeling

INTRODUCTION

Due to their high stiffness and a wide range of load, speed, and operating temperature sustainability, rolling bearings applications have ranged from simple bicycles to very sophisticated gas turbine engines used in aircraft engines and cryogenic turbopumps that form critical parts of the space shuttle propulsion system. At first sight, a rolling bearing indeed appears to be a simple mechanical component consisting of a set of rolling elements rolling between a stationary and rotating race and separated by a cage. Interaction between each one of these elements of the bearing also appears to be quite simple and, therefore, simulation of rolling bearing performance should be a straightforward task. However, it is the coupling between the simple interaction of bearing elements that makes modeling and simulation of real-time performance of a rolling bearing a difficult task. For example, a slight change in shaft speed produces slip between the race and rolling elements; this slip tends to shear the lubricant film at the rolling element to race interface, which produces heat and alters the lubricant temperature and therefore its properties, which in turn affects traction forces between the interacting elements; the change in traction imposes acceleration on the rolling elements, which leads to collision between the rolling elements and cage. Based on the applied loads on the bearing, the loads at each rolling element contact may be different and therefore cage pocket interaction may vary from ball to ball. Furthermore, interaction in one cage pocket may affect interaction in other pockets because the cage is a one-piece element. Thus, although each interaction in the bearing may be quite simple, modeling the bearing as a whole becomes a rather difficult task.

Due to the above complexities in model development, the development of any critical rolling bearing design has often been based on experimental investigations where the actual bearing is tested under simulated conditions to validate the design. Because the number of design and operational parameters is often quite large, such a development process becomes an extremely laborious and expensive task. Such development difficulties have led to analytical model development, where the bearing performance may be simulated parametrically as a function of various design and operational parameters. These simulations can significantly narrow down the number of parameters over which the bearing performance may be critical. Thus, the parameter matrix for experimental investigation can be greatly reduced. In addition, the models are constantly validated against experimental data for increasing confidence in model predictions. Model development tasks have therefore been closely interfaced with experimental investigations. Figure 1 schematically outlines the development process. Based on an available materials database and the pertinent operating conditions, a preliminary design is developed and

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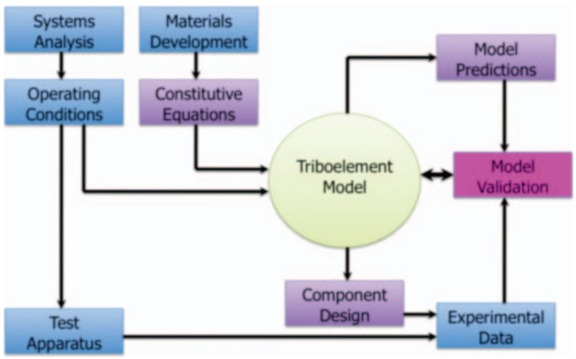

Fig. 1—Typical model development process.

an experimental apparatus, or test rig, is designed. Actual bearing behavior is then measured and compared with the model predictions. Reliability of model predictions therefore increases and the models become increasingly significant for practical design and performance simulation.

Over the past several decades there has been a significant interest in model development, and a number of models have become available for rolling bearing design and performance simulation. A technical review of these models, their capabilities, and limitations is the subject of this article. In addition, anticipated future requirements are outlined to provide guidance for model enhancements and further development.

TYPES OF ROLLING BEARING MODELS

As shown schematically in Fig. 2, fundamentally there are three elements to any triboelement: constitutive equations, geometric compatibility, and governing equations. Constitutive equations define the load deflection relations for the materials being used. For rolling bearings this includes materials for rolling elements, races, and the cage. In addition, the lubricant behavior, as defined by the traction–slip relation, is included under this title. In the event that there is interaction between the races, housing, and shaft, the load deflection relation applicable to both outer race and housing and inner race and shaft must also be included. Rolling bearing geometric compatibility includes any external

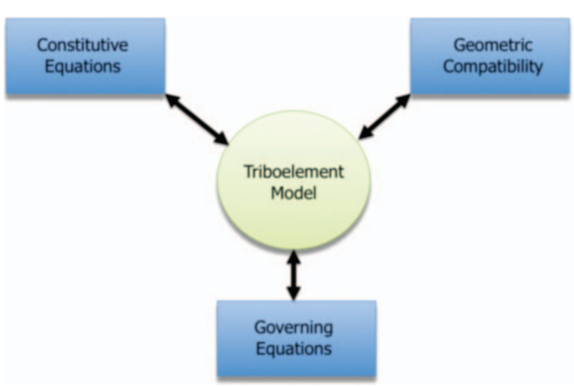

Fig. 2—Basic components of a model.

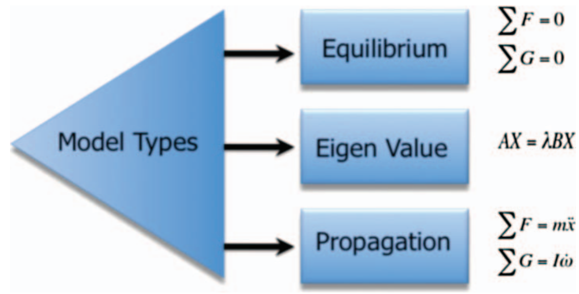

Fig. 3—Types of models and governing equations.

constraints applied on the bearing; a good example is application of preload in a set of two ball bearings that are mounted as a pair, and any axial displacement is constrained for the pair. Governing equations define the laws under which the bearing elements move. This is where the models tend to differ from each other. As shown schematically in Fig. 3, there are basically three types of governing equations: equilibrium, where the summation of all forces and moments are equated to zero; eigenvalue, where equilibrium solutions are sought for certain values of a critical parameter; and propagation or dynamic, where the applied forces and moments are equated to products of mass and acceleration and moment of inertia and angular acceleration, respectively. Thus, there could be three types of models: equilibrium, eigenvalue, and dynamic. For rolling bearings, equilibrium-type models have been used extensively to estimate overall load distribution on the rolling elements, contact stresses, lubricant film thicknesses, bearing stiffness, and classical fatigue life of the bearing. Eigenvalue-type models are used when computation of certain natural frequencies and internal noise generated in the bearing is required. Because stiffness in a rolling bearing is generally nonlinear, pertinent eigenvalue problems represent a linearized simulation in the vicinity of applied operating conditions. A dynamic formulation is required when cage interactions, roller skid and skew, thermal modeling, geometrical imperfections, wear, and all related instabilities have to be modeled. Except for a rather small number of precision applications, where bearing noise is an issue, most rolling bearing applications may be modeled either by a static or a dynamic formulation.

ANALYTICAL OVERVIEW

As discussed above, an equilibrium model consists of force and moment equilibrium equations, which can be solved for compatible displacements, whereas the dynamic model is based on the integration of classical differential equations of motion of each bearing element. Because all cage forces, lubricant traction, and all related frictional forces are quite small in comparison to the applied load support forces at the rolling element–to-race contacts, the equilibrium problems are generally formulated in terms of the applied forces in the axial and radial directions. In the event that there are applied moments on the bearing, which result in relative race misalignment, the moment equilibrium equations are also written, again only in terms of the applied normal forces while neglecting all frictional forces.

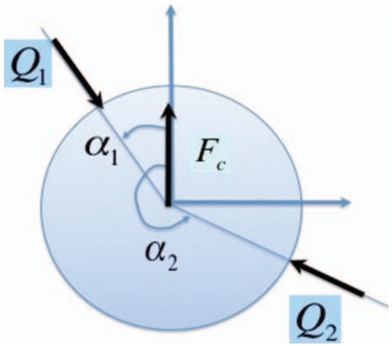

Fig. 4—Schematic of ball loads.

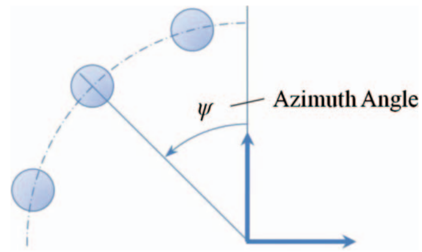

Fig. 5—Ball angular position and race azimuth.

In a dynamic formulation in a generalized six-degrees-of-freedom system, however, all normal and frictional forces are considered. This makes the simulation of the cage motion, which is based on small normal and frictional forces at the various cage contacts, possible. An overview of the analytical formulation, for both the equilibrium and the dynamic models, is presented below.

Equilibrium Modeling in Rolling Bearing

As shown schematically in Fig. 4, the equilibrium equations for a ball, in the axial and radial directions, in an angular contact ball bearing are respectively written as:

$$\sum_{i=1}^{2} Q_i \sin\alpha_i = 0 \quad [1a]$$

$$\sum_{i=1}^{2} Q_i \cos\alpha_i + F_c = 0 \quad [1b]$$

where the subscript i refers to outer and inner race contacts; Q is the contact load; α is the contact angle, and, F_c is the centrifugal force on the rolling element. The contact load is a function of the relative axial and radial positions of the bearing elements. Thus, Eqs. [1a] and [1b] may be reduced to two simultaneous nonlinear algebraic equations in terms of the axial and radial displacements, which may be solved by iterative procedures.

Similarly, the equilibrium equations for the race are written as:

$$\sum_{i=1}^{n} Q_i \cos\alpha_i + F_x = 0 \quad [2a]$$

$$\sum_{i=1}^{n} Q_i \cos\alpha_i \cos\psi_i + F_r = 0 \quad [2b]$$

where n is the number of rolling elements and F_x and F_r are the applied axial and radial forces, respectively, and ψ is the azimuth angle, which defines the angular position of the rolling element around the bearings, as illustrated in Fig. 5.

Once again, Eqs. [2a] and [2b] may be reduced to two algebraic equations in terms of relative axial and radial position of the race, which may be solved by an iterative procedure. Thus, the force equilibrium solution is a two-step process: first, a race position is assumed and the ball equilibrium is solved; race position is then corrected to satisfy the equilibrium equations and ball equilibrium is repeated. The iterative process continues until the race equilibrium equations converge. Such a procedure has a numerical advantage over solving the ball and race equilibrium simultaneously, which, depending on the number of balls in the bearing, may require inversion of a large matrix at each iterative step. Specific details of actual implementation of the solution procedure vary in the different models currently available.

Procedures for roller bearings are similar to that discussed above. The primary difference is in the load displacement relations. For ball bearings, the classical Hertzian point contact solution is used, whereas for roller bearings semi-empirical procedures as outlined by Jones *(1)* and Harris *(2)* are commonly used.

Aside from computing the interacting loads, computation of rolling element velocities compatible with the input race angular velocity is also required. A possible kinematic constraint is that the relative slip between the rolling element and interacting race is zero at a given point in the contact zone. Applying such a constraint at both outer and inner race contacts provides two equations, which are adequate for roller bearings, where the roller just has two velocity components: rotation about its axis and motion of roller center around the bearing. For an angular contact ball bearing, however, the ball angular velocity has two components about the x and z axes, as shown in Fig. 6. In addition, there is ball orbital velocity corresponding to the motion of ball around the bearing. Thus, in addition to the kinematic constraints discussed above, an additional constraint is required to complete the angular velocity formulation. After significant experimental evidence, Jones *(1)* proposed the most commonly used *race control hypothesis*. The hypothesis states that the rolling element angular velocity relative to that of the race about an axis, which is normal to the plane of contact, is zero on the race that provides larger friction torque, and the pertinent race is said to be the *controlling* race. For computation of friction torque, a constant friction coefficient is generally assumed.

In lieu of the race control hypothesis, an alternate constraint may be to minimize the total frictional energy dissipated in the outer and inner race contacts, as stated by Gupta *(3)*. Under any arbitrary lubrication condition, if the frictional energy due to relative rolling element–to-race slip is E_1 and E_2 (at the outer and inner race, respectively) and the inclination of the rolling element angular velocity vector is β, as shown in Fig. 6, then the constraint

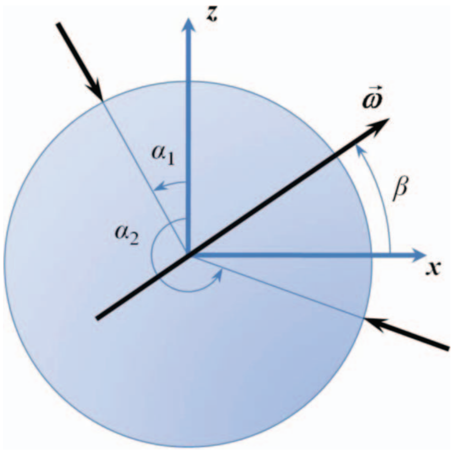

Fig. 6—Orientation of ball angular velocity vector.

is implemented as:

$$\frac{\partial}{\partial \beta}(E_1 + E_2) = 0 \quad [3]$$

Dynamic Modeling in Rolling Bearing

In dynamics modeling, the equilibrium equations are replaced by differential equations of motion, which are integrated as a function of time to obtain a real-time simulation of bearing motion. In general, the bearing element motion is divided into two parts: motion of the element mass center and rotation of the bearing element about its mass center (Walters *(4)*; Gupta *(5)*, *(6)*). Any rolling bearing is comprised of basically four elements: the rolling elements (ball or rollers), the cage, the outer race, and the inner race. The equations of motion for a rolling element are conveniently written in a cylindrical coordinate frame, illustrated in Fig. 7.

$$m\ddot{x} = F_x \quad [4a]$$

$$m\ddot{r} - mr\dot{\theta}^2 = F_r \quad [4b]$$

$$mr\ddot{\theta} + 2m\dot{r}\dot{\theta} = F_\theta \quad [4c]$$

where m mass of the rolling element; (x, r, θ) are the axial, radial, and orbital coordinates; and (F_x, F_r, F_θ) are the components of the applied force vector in the respective directions.

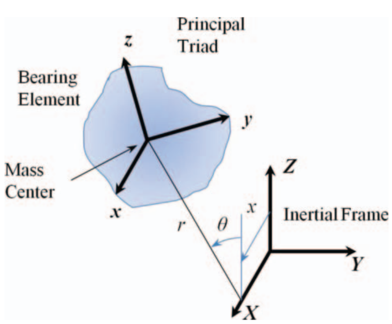

Fig. 7—Cylindrical coordinates for rolling element motion.

Motion of the cage and the races (both outer and inner) may be modeled in the Cartesian coordinate frame (X, Y, Z).

$$m\ddot{x} = F_x \quad [5a]$$

$$m\ddot{x} = F_y \quad [5b]$$

$$m\ddot{x} = F_z \quad [5c]$$

where m is the mass of the element being considered and (F_x, F_r, F_z) are components of the applied force vector in the (X, Y, Z) coordinate frame.

In the most generalized fashion the rotational motion on any bearing element may be modeled by the classical Euler equations of motion, as outlined by Walters *(4)* and Gupta *(5)*, *(6)*, written in a body fixed frame, located along the principal triad (oriented along the three principal axes of the element), as shown in Fig. 7.

$$I_1\dot{\omega}_1 - (I_2 - I_3)\,\omega_2\omega_3 = G_1 \quad [6a]$$

$$I_2\dot{\omega}_2 - (I_3 - I_1)\,\omega_3\omega_1 = G_2 \quad [6b]$$

$$I_3\dot{\omega}_3 - (I_1 - I_2)\,\omega_1\omega_2 = G_3 \quad [6c]$$

where (I_1, I_2, I_3) are the three principal moments of inertia, $(\omega_1, \omega_2, \omega_3)$ are the three components of the angular velocity vector, and (G_1, G_2, G_3) are the three components of the applied moment vector.

Each second-order differential equation in the above equations can be reduced to two first-order equations by introducing velocity as an additional variable. For example, Eq. [5a] may be written as:

$$\dot{x} = v \quad [7a]$$

$$m\dot{v} = F \quad [7b]$$

Likewise, the angular velocities and acceleration may be reduced to angles and their first derivatives. In general, three angles, similar to classical Euler angles, corresponding to the three angular velocities can be defined as done by Walters *(4)* and Gupta *(5)*, *(6)*. In addition, the angles may be used to define transformation between different coordinate frames while solving for geometrical interaction between two elements. These definitions are slight variations of Euler angles such that no singularities are encountered for range of motion of the bearing elements and the transformation matrix remains orthogonal.

In order to facilitate efficient implementation and vectorization on a computer system, the equations of motion may be presented in matrix form in terms of generalized position and derivative vectors defined by the three coordinates locating the mass center of a bearing element and three angles locating the angular position. Such a generalized position and derivative vectors may be written as:

$$\boldsymbol{x} = \left\{x, r, \theta, \dot{x}, \dot{r}, \dot{\theta}, \eta, \xi, \zeta, \dot{\eta}, \dot{\xi}, \dot{\zeta}\right\}^T \quad [8]$$

$$\boldsymbol{y} = \dot{\boldsymbol{x}} = \left\{\dot{x}, \dot{r}, \dot{\theta}, \ddot{x}, \ddot{r}, \ddot{\theta}, \dot{\eta}, \dot{\xi}, \dot{\zeta}, \ddot{\eta}, \ddot{\xi}, \ddot{\zeta}\right\}^T \quad [9]$$

where $\boldsymbol{x}$ and $\boldsymbol{y}$ are, respectively, the generalized position and derivative vectors and (η, ξ, ζ) are the three transformation angles. The applied force components from Eq. [4] are substituted for the mass center acceleration components in Eq. [9], and the angular accelerations are computed in Eqs. [6] from the applied moments. These angular accelerations are then combined with the derivative of matrix equation establishing the relationship between the angular velocity and rate of change of transformation angles (Gupta *(6)*) to arrive at the values of second derivative of the transformation angles in Eq. [9].

The above vectors are written in terms of polar components for a single rolling element. The vector length can be expanded to include a set of 12 equations for each rolling element. For the cage and races, similar components may be defined in a Cartesian system. The set of first-order equations can then be numerically integrated to obtain a generalized real-time dynamic simulation of rolling bearing performance.

Computation of applied forces and moments is divided into three parts: normal forces, friction or traction forces, and then computing moments about the element mass center generated by the interacting forces. Normal forces are computed by a straightforward geometric interaction analysis, which determines the geometric interaction between the interacting elements and then uses a force deflection relation, as outlined by Harris *(2)*, to compute the normal force. Similarly, relative velocity or slip vectors can be derived from the velocities of the interacting elements and then a traction–slip relation may be used to compute the applied traction forces (Gupta *(6)*). Once the forces are determined, the applied moments are a simple cross-product of the position vector locating the point of interaction relative to element mass center and the computed normal and traction forces. A systematic procedure for this computation is outlined in Fig. 8. Basically, from the position vectors of two interacting elements A and B, a relative position is computed. Then the geometry of the interacting elements is subtracted to compute the geometrical interaction. This geometrical interaction essentially represents the elastic deformation of the interacting elements that are contacting each other. Knowing the elastic deformation, a load deformation model, such as a Hertzian point contact or a similar line contact model (Harris *(2)*), is used to compute the applied loads. Similarly, from the relative velocity of the interacting elements a slip component, tangential to the plane of contact, is computed. This, along with the contact stress corresponding to the computed normal load, is input into a lubricant traction model to compute a traction coefficient, which is a ratio of the traction (or friction) force to the applied normal load. Thus, the applied traction force is computed. Finally, a cross-product of position vectors locating the point of interaction and the applied load vectors yields the applied moment.

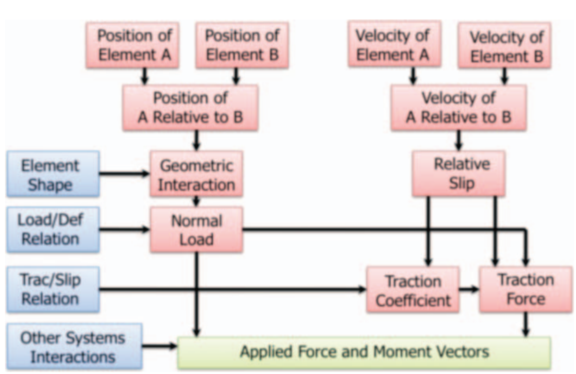

Fig. 8—Generic architecture of interaction model and applied load computation.

Perhaps the most important parameter that controls the dynamic behavior of a rolling bearing is traction between interacting bearing elements. The subject has been the main driver for extensive research in modeling frictional interactions, lubricant film formation, and elastohydrodynamic lubrication over the last several decades. It has been well established that in a concentrated rolling sliding contact the traction coefficient is dependent on relative sliding or slip velocity between the interacting elements. In addition, the lubricant properties and rheology play a dominant role. In a simplistic fashion, Gupta *(6)* has promoted a simple hypothetical model, based on the early works of Kragelskii *(7)*. Based on experimentally observed traction/slip behavior of a wide range of liquid and solid lubricants, the following algebraic equation is proposed to define the traction coefficient as a function of slip velocity:

$$\kappa = (A + Bu)\exp(-Cu) + D \qquad [10]$$

where κ is the traction coefficient at a slip velocity, u, and A, B, C, and D are empirical constants, normally derived from experimental traction data.

This simplified model works very well for most solid lubricants, where the traction coefficient is relatively insensitive to rolling velocity and contact pressures. It can also be used for liquid lubricants at a given rolling velocity and contact pressure when the model coefficients are fitted to experimentally observed behavior.

More realistic and sophisticated models are available from the past work on lubricant film formation and the mechanics of elastohydrodynamic lubrication. There are really two physical mechanisms that are taking place in a rolling/sliding contact. First, a lubricant film is formed by hydrodynamic action between two elastically deformed surfaces under the applied contact pressure. The lubricant is then sheared in the contact zone and a traction force results. Thus, computation of traction consists of two parts: computation of the film thickness and then solution of the combined thermal and mechanical problem in the contact zone to compute traction. The lubricant hydrodynamic equations have been solved for both line and point contacts and the solutions are curve-fitted to readily usable formulae (Cheng and Sternlicht *(8)*; Cheng *(9)*; Hamrock and Dowson *(10)*). For modeling traction, a somewhat simplified approach is to assume Newtonian behavior of the lubricant in the high-pressure contact region and compute traction by solving the flow equation with prescribed surface velocities and temperatures (Kannel and Walowit *(11)*). The model is based on the following equations:

$$\text{Energy equation: } K\frac{\partial^2 T}{\partial z^2} = -\tau\dot{s} \qquad [11a]$$

$$\text{Geometric compatibility: } \frac{\partial u}{\partial z} = \dot{s}(\tau, p, T) \qquad [11b]$$

$$\text{Constitutive equation: } \dot{s}(\tau, p, T) = \frac{\tau}{\mu(p, T)} \qquad [11c]$$

where K is thermal conductivity, T is absolute temperature, τ is shear stress, $\dot{s}$ is the shear strain rate, μ is lubricant viscosity, p is pressure, u is the lubricant velocity, and z is the coordinate across the film.

The above equations are solved for shear stress and temperature distribution across the film; the shear stress is then integrated over the contact area to compute the total traction force. The primary input to the model is the viscosity–pressure–temperature relation in Eq. [11c].

In the Newtonian model, the viscosity varies exponentially as a function of both pressure and temperature. Thus, at very high pressures the Newtonian model yields extremely high viscosities, such that the lubricant tends to behave as a solid rather than as a fluid. Under such conditions, the Newtonian model may yield unrealistic traction behavior. To better simulate the lubricant behavior under such conditions, a viscoelastic model has been proposed (Johnson and Tevaarwerk *(12)*; Bair and Winer *(13)*). In such models a shear stress/strain rate equation is introduced in addition to the viscosity relation used in the Newtonian model:

$$\dot{s} = \frac{1}{G}\frac{\partial \tau}{\partial t} + \frac{\tau_o}{\mu} f\left(\frac{\tau}{\tau_o}\right) \qquad [12]$$

Here G is the shear modulus, τ is the shear stress, and τ_o is defined as a critical shear stress. Two forms of the shear stress functions have been proposed (Johson and Tevaarwerk *(12)*; Bair and Winer *(13)*):

$$f\left(\frac{\tau}{\tau_o}\right) = \sinh\left(\frac{\tau}{\tau_o}\right) \qquad [13a]$$

$$f\left(\frac{\tau}{\tau_o}\right) = \tanh^{-1}\left(\frac{\tau}{\tau_o}\right) \qquad [13b]$$

Computation of shear stress distribution through the lubricant film with the viscoelastic model requires integration of a differential equation (Gupta, et al. *(14)*), whereas the Newtonian model may be implemented almost in closed form (Gupta *(6)*). In either model there are three model coefficients: reference viscosity, pressure–viscosity, and temperature–viscosity coefficients in the Newtonian model, and effective viscosity, shear modulus, and critical shear stress in the viscoelastic model. Because actual measurement of these constants is extremely difficult, they are generally derived by regression analysis of experimental traction data (Gupta, et al. *(14)*, *(15)*). A very extensive review of elastohydrodynamic lubrication, the associated properties, and their measurement was presented by Jacobson *(16)*.

For modeling the performance of rolling bearings, both of the above models work fairly well for a wide range of operating conditions. Under very high contact pressures and high speeds, however, the Newtonian model demonstrate a very high traction slope in the low-slip region, the validity of which has been controversial; this high traction slope also results in some numerical difficulties when integrating the equations of motion. The viscoelastic model is generally free of such problems, but it does require significantly more computing effort because a differential equation has to be solved through the lubricant film for each contact in the bearing at each time step. The continually increasing computing speed of modern computing systems has certainly helped in executing very computationally intensive models in reasonable lengths of time.

In more recent works, Larsson, et al. *(17)*, *(18)* have experimentally measured some of the lubricant properties, which enter in the computation of lubrication traction, for a number of commonly used lubricants. The experimental investigations include measurement of viscosity, pressure–viscosity coefficient using a high-pressure viscometer, temperature–viscosity coefficient, bulk modulus, thermal conductivity, and heat capacity. The measured properties are presented in terms of readily usable empirical relations as a function of operating conditions.

Development History

Computer modeling of rolling bearings dates back to the 1960s when digital computing was becoming a reality with popularity of large mainframe computing systems. Perhaps the first of the computer codes to carry out analysis of a rolling bearing may be credited to Jones *(1)*, who implemented his analysis in a computer code to predict load distribution, stiffness, and, most important, fatigue life of a rolling bearing. The Jones *(1)* computer code served as a primary tool for the design of rolling bearings. This work was based on static equilibrium formulation; the centrifugal forces and gyroscopic moments encountered at high speeds were added as additional external forces and moments. The codes were therefore called *quasi-static*. Because fatigue was the primary mode of failure for most rolling bearings at the time, the Jones *(1)* code served the industry well. Somewhat in parallel with Jones's *(1)* work, Harris *(2)* also promoted quasi-static modeling of rolling bearings and presented a fairly rigorous analysis of interaction between rolling elements. Also, Poplawski and Mauriello *(19)* pursued modeling of skidding in lightly loaded ball bearings. Just a few years later, Harris *(20)*, *(21)* presented substantial work on frictional interactions at the rolling element to race interface and skidding in ball bearings. In parallel, Poplawski *(22)* extended his work on ball bearings to roller bearings, where an attempt to model cage forces was also made.

With the continued advancement in the computer industry, the 1970s started with a major thrust on computer modeling of rolling bearing performance. Walters *(4)* presented a generalized dynamics model to solve the differential equations of motion of the cage in an angular contact ball bearing with constrained ball motion and the model was implemented in a computer code, BASDAP. Later, Gupta *(23)* extended this work to generalize the ball motion with complete six degrees of freedom. Based on quasi-static models, Mauriello, et al. *(24)* presented a somewhat simplified cage analysis. Rumbarger *(25)* modeled cage deformation as a function of thermal expansion and quasi-static motion of rollers in a high-speed roller bearing. Perhaps the most significant advancement of quasi-static models is credited to the work of Crecelius and Privics *(26)* at SKF and the related publication of a very extensive computer code, SHABERTH, which incorporated simultaneous operation of several bearings; in addition, transient thermal analysis was also incorporated with the objective of presenting a complete mechanical and thermal systems tool for rolling bearing design. SHABERTH is still widely used in the industry, as discussed later. In the dynamics area, Kannel and Bupara *(27)*, based on the earlier work of Walters *(4)*, further investigated the dynamics of cage motion and its coupling with elastohydrodynamic lubrication at the ball–race contacts. With continued interest in modeling real-time dynamic effects in rolling bearing, Gupta *(28)* integrated the generalized ball motion (Gupta *(23)*) with the cage differential equations of motion; in addition, cylindrical roller bearings were included in the model along with the angular contact ball bearings; the work was published as a generalized dynamics tool for both ball cylindrical roller bearing, Dynamics of Rolling Element Bearings (DREB). Soon after implementing this generalized model to practical problems, the required computing effort imposed a severe strain. After investigating the detailed frequency response of a rolling bearing, Gupta *(29)* introduced a time-varying equilibrium constraint to filter out the very high-frequency components and thereby provide a substantially increased time step size; the work was implemented as a faster version of DREB, RAPIDREB. RAPIDREB provided bearing performance simulations over a greatly increased time domain, as required for graphics animation of bearing element motion. Soon after the DREB development, Brown, et al. *(30)* undertook a task to develop a roller bearing dynamics code, TRIBO1, especially for cylindrical roller bearings in a gas turbine engine, and Conry *(31)* worked on a dynamics model for lightly loaded cylindrical roller bearings.

Further development of quasi-static models continued at SKF with the work of Ragen *(32)*, who incorporated gears in the quasi-static systems code and published an extended code, TRANSIM. More specialized versions of computer codes aimed at modeling cylindrical roller bearings include CYBEAN, published by Kleckner, et al. *(33)*, and SPHERBEAN for spherical roller bearings by Kleckner and Privics *(34)*, also at SKF.

Advancement in computing hardware continued rapidly in the 1980s when personal computers appeared and demonstrated significant potential for scientific computing. Also, advancements in supercomputers were significant; vectorization of sophisticated computationally intensive codes resulted in significant reduction in computing effort. With such advancements in computing hardware and processing speed, Gupta *(6)* further generalized his formulations for rolling bearing performance simulations and published a completely new code, Advanced Dynamics of Rolling Elements (ADORE). Faster computing permitted integration of the equations of motion over larger time intervals, which led to better insight into the rolling/sliding interactions and simulation of wear in rolling bearings (Gupta and Forster *(35)*). The geometrical generalizations, on the other hand, permitted modeling of geometrical imperfections and optimization of manufacturing tolerances (Gupta *(36)*, *(37)*). In addition to ball and cylindrical roller bearings, the geometrical generalization led to the simulation of tapered roller bearings (Gupta *(38)*). In parallel with the above work, Meeks and Karen *(39)* and Meeks *(40)* developed models for ball bearing cage, or separator, dynamics and published a code, SEPDYN. In the quasi-static area, Sague *(41)* published a code, PREBES, and Poplawski *(42)* published a computer model, COBRA.

Along with the advancement in computer processors, the substantial advancement in materials manufacturing technology contributed to a notable enhancement of the fatigue life of rolling bearings. As a result, higher speeds, loads, and temperatures became more practical for rolling bearings, and this led to an increased interest in real-time dynamics models for rolling bearing performance simulation. With due recognition and limitation of the Newtonian models to model traction in rolling/sliding contacts, Gupta *(43)* incorporated a viscoelastic model in the bearing dynamics code, ADORE. This could only be possible by the advancing computing speed, because the model requires rather extensive computing in each contact in the bearing. With the real-time simulation of a bearing overall several shaft revolution now becoming a routine, Gupta *(44)* took the simulated generalized motions of bearing elements, assembled them in an animated display of bearing element motion, and presented a sister code, Animated Graphics of Rolling Elements (AGORE). Meeks, et al. *(45)* and Meeks and Polendo *(46)* also made advancements of the code SEPDYN and published the codes BASDREL and BABERDYN. Aramaki *(47)* used the quasi-static equilibrium equations along with approximated acceleration on the rollers to simulate roller slippage in significantly reduced computing effort.

In more recent years, advancement in rolling bearing dynamics modeling continued with varying approaches. Stacke, et al. *(48)* and Stacke and Fritzson *(49)* at SKF used multibody techniques, with particular emphasis on contact problems between bearing elements, to model overall dynamic behavior of rolling bearings. This resulted in the computer model BEAring Simulation Tool (BEAST). Similar to other dynamic models, BEAST integrates a set of differential equations to obtain real-time simulation of bearing performance. By inserting multiple spring and dashpot elements, BEAST generates appropriate transfer functions to model bearing interaction with external system. Presently, BEAST is used internally within SKF for bearing dynamics performance simulation. Gupta *(50)* undertook a major initiative to model thermal interactions between bearing elements and further advanced the bearing model ADORE for the modeling effect of thermal interactions in overall bearing dynamics. The heat generated at the various interactions is used to compute a temperature field, which affects bearing geometry, which in turn alters the interacting loads and dynamics of bearing elements. Based on the works of Nakhimovski *(51)*, *(52)*, BEAST was also advanced to include thermal interactions and flexibility in bearing elements, such as the cage and the bearing races. A more recent version of BEAST and the associated multibody approach was discussed by Ioannides, et al. *(53)*. Ghaisas, et al. *(54)* examined the role of cage pocket clearances in cylindrical roller bearings and demonstrated how the cage forces and resulting cage motion are altered as the pocket clearances increase.

Modeling the effect of surface defects on rolling bearing dynamics is a relatively new area of interest. For some critical applications the time to bearing failure once a fatigue spall or other defect develops at the race surface has been of significant interest in avoiding catastrophic situations. Gupta *(55)* used the arbitrary geometry features in ADORE to model raceway defects as a varying race surface radius with a prescribed defect shape and size. The defect geometry is thereby correlated to overall bearing dynamics. Ashtekar, et al. *(56)* used a modified force displacement relation, corresponding to the defect geometry, to compute the applied forces while integrating the differential equations of motion; their work also provided an excellent review of the available literature on this subject. Fritzson, et al. *(57)* have also presented modifications to the contact mechanics model to model surface defects, as induced by fretting and fatigue. Modeling cage flexibility while integrating the equations of motions of the bearing elements is the subject of the very recent work by Weinzapfel and Sadeghi *(58)*. A Newmark-type implicit integration method combined with a Newton-Raphson iterative technique has been recently used by Leblanc, et al. *(59)* to model the dynamics of a roller bearing with flexible races. With particular emphasis on prediction of rolling element slip and the resulting cage forces in planetary application of rolling bearings, Houpert *(60)*, *(61)* has published a code, CAGEDYN, to model overall bearing dynamics.

Increasing interest in modeling real-time dynamics of rolling bearing is clearly evident from the available literature over the last several of decades and continued publication of new material. The original classical work of Harris *(2)* is now in its fifth edition (Harris and Kotzalas *(62)*, *(63))*. Practical implementation of very sophisticated mathematical techniques is gradually becoming a reality as the available computing power continually unfolds.

Presently Available Codes

Though the results of a large number of development activities are publicly available as research papers and reports, the related software tools fall into two categories: codes that are developed and used internally within an organization and codes that are commercially available as software tools for modeling the performance of rolling bearings. Indeed, a lot of technology and expertise resides within the rolling bearings manufacturing companies, bearing designers, and bearing users such as gas turbine engine manufacturers and a wide range of aerospace organizations. These codes offer practical implementation of years of development experience and expertise to a broad range of applications. The organizations use the codes internally for application support and development of new products. In the area of commercially available tools, a number of computer codes have been packaged with reasonably easy to use input–output interfaces and they are offered as commercial software products. A brief review of some of these tools is presented below.

A. B. Jones Bearing Analysis Software

The A. B. Jones Bearing Analysis Software, originally published and marketed in the 1960s, is still used fairly widely for bearing design in the industry. Perhaps the original codes, along with some variations, are still available.

SHABERTH

SHABERTH is a very comprehensive systems level quasi-static model. It was originally developed by Crecelius and Pirvics *(26)* at SKF under contract with the U.S. Air Force. The code performs a detailed quasi-static analysis for a multiple bearing and shaft system, including some thermal effects. Because the original version of the code was in the public domain, varied versions of the code have also been developed and made available by different organizations.

COBRA

COBRA is a multi-bearing quasi-static systems model. It can model a number of rolling bearings simultaneously with either a rigid or flexible shaft. The code can interface with the finite element code ANSYS to integrate race fit analysis for both interference fits and thermal gradients. The program has been developed by Poplawski *(42)* and it is presently marketed by J. V. Poplawski & Associates. Information of the code is readily available *(67)*.

ADORE

ADORE, developed by Gupta *(6)*, is a fully dynamic model. The classical differential equations of motion are integrated as a function of time to provide real-time dynamic performance simulation of rolling bearings, including ball, cylindrical, and tapered roller bearings. The code also includes a quasi-static module, which is used to compute the initial conditions when integrating the differential equations of motion. The code is marketed by Pradeep K. Gupta Inc., and detailed information is available *(68)*.

Current Limitations and Future Requirements

Lubricant Traction

Lubricant traction in a rolling/sliding contact has been identified as a key parameter that controls the dynamic behavior of a rolling bearing. Thus, realistic input to model the lubricant accurately is a key to reliable prediction of bearing performance. Due to rather complex behavior of the lubricant as a function of operating temperatures and pressures, the effective lubricant properties are generally derived by back-fitting a model to actual experimental data via a regression analysis. Because these traction experiments require specialized test rigs (Wedevan *(64))*, the number of lubricants for which such data are available is few. To cover a wider application domain, it is essential to expand these databases for both fully formulated lubricating oils and the base oils used in greases. In addition to modeling the base oil in greases, other effects that contribute to traction and resistance to rolling element motion have yet to be explored.

Data on solid lubricants is even more limited in comparison to liquid lubricants. Thus, simulation of solid lubricated rolling bearings is greatly restricted. With the recent innovation of ceramic rolling bearings, the development of traction data for solid lubricants and coated and unlubricated surfaces is a key to realistic modeling of these advanced bearing concepts.

Thermal Interactions

Although the heat generated in each individual contact in a rolling bearing can be computed fairly well once the traction or friction behavior in the contact is defined, most models are limited in modeling the transfer of this heat to rest of the system. Perhaps an integration of the thermal finite element steady-state and/or transient models with the bearing dynamics model is essential to model the combined mechanical and thermal behavior of a rolling bearing. A large difference in timescales for the thermal and mechanical problems makes this a numerically difficult task.

Churning and Drag Effects

In oil-lubricated bearings filled with circulating lubricant, there is a substantial drag and churning moment applied on the bearing elements. A realistic simulation of these effects is essential for modeling both the applied forces and moments on the bearing elements, as required in the equations of motion, and in the computation of overall power loss in the bearing. In most of the current bearing models the churning and drag models are still based on the very simple models (Rumbarger, et al. *(65))* based on classical laminar and turbulent flows. Modern innovative techniques of computational fluid dynamics (CFD) may be applied to improve these models and therefore significantly enhance the dynamic simulation of a rolling bearing.

Code Language and Architecture

Most of the software tools for engineering computation used today still employ FORTRAN language, the support for which has substantially declined over the last decade. Furthermore, with the advent of modern object-oriented languages, such as C++ and Java, most universities have withdrawn FORTRAN courses from their academic curriculum. As a result, new engineers entering the industry find it very difficult to work with FORTRAN codes. Certain floating point processing and manipulation of multidimensional arrays are perhaps the two key elements that make FORTRAN a very efficient language for engineering computation. Although there has been some effort in the development of such capabilities for the modern computer languages, the current limitations are viewed as significant constraints for the scientific computing community. On the positive side, the object-oriented languages make graphic processing and network computing extremely easy and powerful. Perhaps an effort to perform some of the computing tasks, unique to FORTRAN, by innovative software concepts in the modern languages may be worthwhile until the standards for the modern languages are expanded to better accommodate the engineering and scientific communities. A very recent initiative by DynaTech Engineering *(66)* is one example of an attempt to use Java for engineering computing related to rolling bearing modeling.

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