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Vibrational Characteristics of Ball Bearings

The classical differential equations of motion of the ball mass center in an angular contact thrust loaded ball bearing are integrated with prescribed initial conditions in order to simulate the natural high frequency vibrational characteristics of the general motion. Two distinct frequencies are identified in the analytical simulation and their existence is also confirmed experimentally. One of the frequencies is found to be associated with the Hertzian contact spring at the ball race contact and it is therefore defined as the "elastic contact frequency," Ω_e. The other dominant frequency corresponding to oscillatory motion of the ball in the raceway groove appears to be kinematic in nature and it is, therefore, termed as the "bearing kinematic frequency," Ω_k. It is shown that for a given bearing Ω_e and Ω_k vary as, respectively, 1/6 and 1/2 powers of the ball contact load and, therefore, for a given load these frequencies correspond to the natural frequencies of the bearing as applied in any vibrational analysis or simulation.

Introduction

Primarily due to the ease in the measurement of high frequency vibrations generated within a rolling-element bearing a strong interest has been recently generated in correlating bearing performance to the frequency contents of a signal picked up by a accelerometer mounted on the stationary race. To a substantial degree of success such techniques have been used in detecting defects and diagnosing failures in bearings [1–4].[1] The procedure generally consists of obtaining the characteristics of an undamaged bearing and comparing these characteristics with those of a possibly damaged bearing. The low frequency vibrations generated by certain bearing defects have been found to modulate the amplitudes of some characteristic natural frequencies of the bearing and when these natural frequencies are known a proper demodulation of the signal provides substantial insight into the bearing behavior. Thus characterization of the bearing natural frequencies has become an important task in this new developing area in rolling bearing technology.

The complexities associated with various types of interaction between rolling elements, races and cage have greatly restricted the investigations of real time dynamic simulation of the rolling element motion. Recently, Gupta [5] formulated the generalized ball motion in terms of the classical differential equations of motion allowing the ball mass center to translate arbitrarily in space along with generalized rotations about the mass center. Such a formulation has resulted in a complete simulation of ball motion which is needed for the determination of any characteristic vibration frequencies associated with the bearing.

One of the primary objectives of this paper is to investigate the high frequency motion as simulated by the dynamic formulation due to Gupta [5] and correlate the natural frequencies with the bearing parameters. An experimental verification of the analytically predicted frequencies is also undertaken.

Analytical Simulations

As shown in Fig. 1, the general motion of the ball mass center can be expressed in cylindrical coordinates by the following differential equations of motion

$$\begin{aligned} m\ddot{x} &= F_x \\ m\ddot{r} - mr\dot{\theta}^2 &= F_r \\ mr\ddot{\theta} + 2m\dot{r}\dot{\theta} &= F_\theta \end{aligned} \tag{1}$$

where m is the mass of the ball.

The applied force components F_x, F_r, and F_θ are determined by the ball race contacts loads and tractions. In simulating any contact vi-

[1] Numbers in brackets designate References at end of paper.

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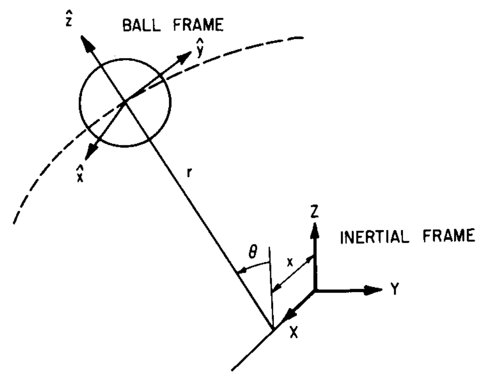

Fig. 1 Ball coordinate frames

brations between the ball and races the orbital motion can be disregarded and only x and r components will be relevant. Also, since the contributions due to tractive forces are primarily along the θ direction and those along x- or r-directions are negligible compared to the normal contact loads, the tractive forces will not have any significant effect on the ball accelerations along the x- and r-directions.

In terms of the nondimensional variables defined earlier by Gupta [5], the equations of motion along the x- and r-directions are written as

$$\frac{d^2X}{d\tau^2} = F_x{}^*$$

$$\frac{d^2R}{d\tau^2} - (R + Re)\left(\frac{d\theta}{d\tau}\right)^2 = F_r{}^* \quad (2)$$

where $X = x/r_0$; $R = r - r_e/r_0$; $Re = r_e/r_0$; $\tau = t\sqrt{Q_0/mr_0}$; $F_x{}^* = F_x/Q_0$; $F_r{}^* = F_r/Q_0$ and r_0, r_e and Q_0 are, respectively, the ball radius, pitch radius and static contact load at the ball race contact.

For any specified position vector, locating the ball mass center relative to the races, the normal contact loads, and, therefore, $F_x{}^*$ and $F_r{}^*$, are computed using the classical Hertz contact theory. In case of static equilibrium clearly the accelerations will be zero, but if the bearing is disturbed from the equilibrium position, the ball mass center will move according to equation (2). If any natural frequencies are associated with such a motion, then they should be clearly evident by a somewhat more detailed form of equation (2). Unfortunately, the expressions for the contact forces are greatly complicated by the geometry of the bearing and relevant kinematic features determining the ball race contact mechanics in an angular contact ball bearing. However, an analysis in which the inner race is held in static equilibrium with the applied thrust load and the outer race is fixed in space has been presented by Gupta [5] and a computerized simulation corresponding to the analytical formulation is also readily available. In the present investigation, therefore, this computerized simulation is used to solve equation (2) numerically when the initial conditions are set close to the equilibrium conditions and contacts are basically assumed frictionless while the initial angular velocities are determined simply by kinematics when the race speed is prescribed. The particular bearing used in the present investigation is selected from the experimental work carried out earlier by Winn [3]. This bearing will be denoted as Bearing A in the present investigations and the relevant bearing data is

Ball diameter	= 12.7 mm (0.50 in.)
Pitch diameter	= 70 mm (2.7557 in.)
Contact angle	= 30°
Outer race curvature factor	= 0.52
Inner race curvature factor	= 0.515
Number of balls	= 14

When a thrust load of 2224 N (500 lb) is applied and the inner race is subjected to an angular velocity of 1000 rpm, while the outer race is held stationary, the axial and radial acceleration components, as given by equation (2), obtained by the computerized simulation of Gupta [5] are shown in Fig. 2. Fig. 2 shows the corresponding components of the position vector locating the ball mass center. It is clearly seen that the accelerations have two distinct frequencies. Furthermore, the X and R components of acceleration corresponding to the high frequency are in phase while the low frequency contributions are 180 deg out of phase. It is true that under ideal conditions this vibratory motion may be damped out due to friction but in practical applications, where the bearing is constantly subjected to subtle disturbances this type of high frequency motion has been found to exist under steady-state conditions. A detailed understanding of this motion, therefore, has a substantial practical relevance.

In the mode when the ball indents the race it is clear that both X and R components must be in phase. Normal contact vibration over the Hertzian spring, as will be discussed in somewhat more detail later, is, therefore, expected to result in the high frequency motion shown in Fig. 3. The mechanism for the lower frequency is not clear in a straightforward fashion. However, if the X and R components are 180° out of phase then the variations in contact angles at the outer and inner race contacts should be 180 deg out of phase. Furthermore, since the bearing has a pure thrust load and the inner race is held in static equilibrium, the contact loads should vary such that equilibrium of the inner race is satisfied with the varying contact angles. All such variations of contact angles and loads are shown in Fig. 3. It is also seen that only the contact load at the outer race contact has the high frequency content. This is primarily due to the assumption that the inner race is always held in static equilibrium. Thus the contact spring at the inner race contact basically applies a static force which is in equilibrium with the applied thrust load.

The existence of these two distinct frequencies of the natural motion has also been found experimentally. However, before discussing the available data a correlation of these frequencies with some bearing parameters will be helpful for engineering purposes.

Natural Frequencies of Ball Motion

From the general characteristics of the ball motion discussed above the two natural frequencies may be correlated, respectively, with the Hertzian elastic spring and the rigid body type oscillatory motion of

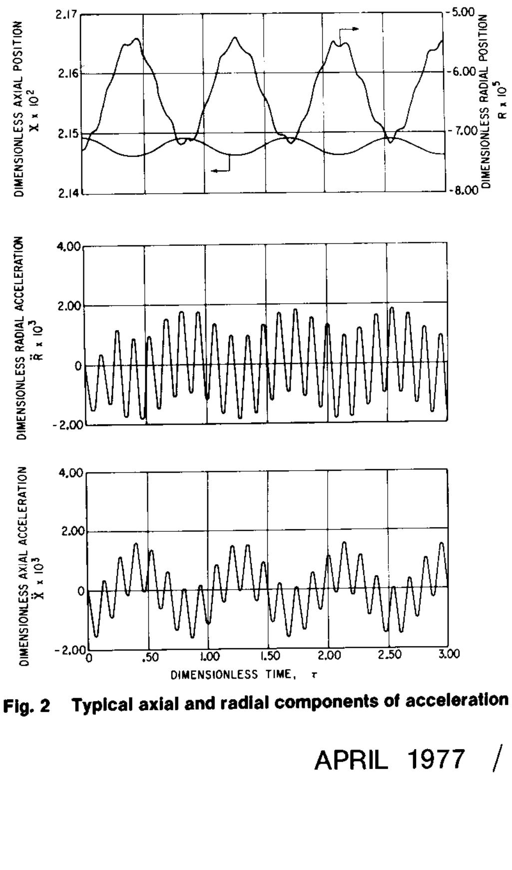

Fig. 2 Typical axial and radial components of acceleration

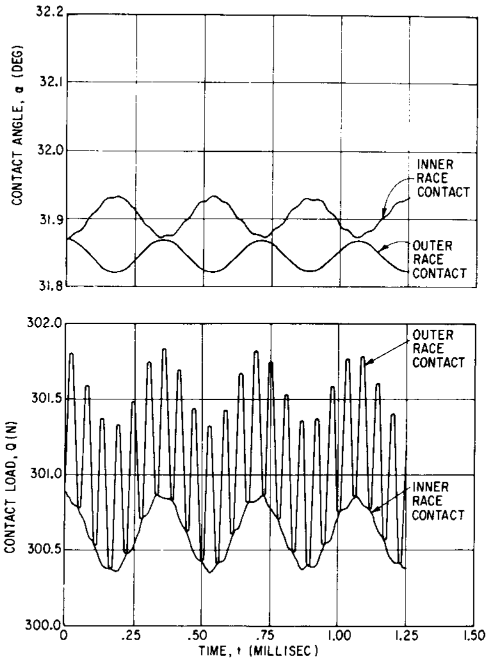

Fig. 3 Variations of computed elastic contact frequency, ω_e and bearing kinematic frequency, ω_k as a function of ball-race contact load

the ball. Strictly for the purpose of nomenclature these frequencies are, therefore, called "elastic contact frequency," Ω_e and "bearing kinematic frequency," Ω_k.

(1) Elastic Contact Frequency, Ω_e. The load deflection relationship for elastic point contact between two interacting bodies is given by the classical Hertzian theory as

$$Q = C\delta^{3/2} \tag{3}$$

where Q is the load, δ is the deflection and C is a constant dependent on elastic properties and geometry of the bodies.

When the vibratory motions of the ball is such that the variation in the contact deflection is small compared to a nominal deflection, then equation (3) can be differentiated to obtain an equivalent stiffness,

$$\frac{\partial Q}{\partial \delta} = \frac{3}{2} C\delta^{1/2} = \frac{3}{2}\frac{Q}{\delta} \tag{4}$$

The natural frequency of ball motion corresponding to the above

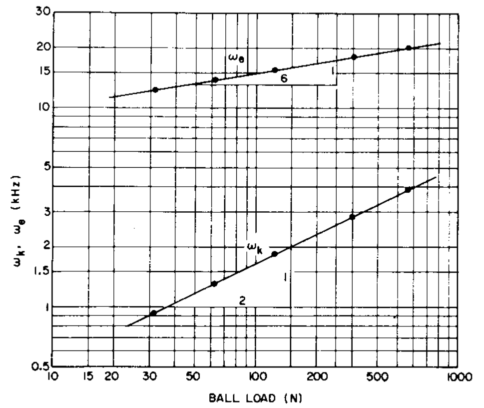

Fig. 4 Variations of computed elastic contact frequency, ω_e and bearing kinetic frequency, ω_k as a function of ball-race contact load

stiffness is thus

$$\Omega_e = \frac{1}{2\pi}\sqrt{\frac{3Q}{2m\delta}} \tag{5}$$

where m is the ball mass.

In the analytical simulations presented earlier, Ω_e will be relevant only for the outer race contact, since the inner race contact spring just provides a static force. This means that the inner race is considered to be massless. In practical systems where the inner race may not move instantly to satisfy equilibrium, a spring similar to the outer race contact will also exist at the inner race contact. Clearly, these two springs will be in parallel and if subscripts 1 and 2 are used, respectively, for the outer and inner race contacts then the expression for Ω_e may be written as:

$$\Omega_e = \frac{1}{2\pi}\sqrt{\frac{3}{2m}\left(\frac{Q_1}{\delta_1} + \frac{Q_2}{\delta_2}\right)} \tag{6}$$

By combining equations (3) and (5) it will be clear that Ω_e varies as $1/6$ power and load. The high frequency present in the ball motion correlates extremely well with Ω_e. From the analytical simulation the estimated high frequency ω_e is plotted in Fig. 4 as a function of ball load Q and the $1/6$ power variation is clearly seen. It is also found that ω_e agrees closely with Ω_e to about 2 percent.

It may be noted from Fig. 4 that at large contact loads or at very high speeds, Ω_e and Ω_k will be comparable with each other and a clear distinction between these two frequencies, as seen in Fig. 3, may not be seen. Although the analysis presented by Gupta [5] is valid for any loads and speeds, when the inner race is assumed to be massless, the simple correlations presented above are only relevant to low speed conditions, when the contact angles at the inner and outer raceways are closely identical. The primary reason for this restriction in the present investigation is simply the nonavailability of high frequency ball motion data in very high speed bearings. It is expected that as more experimental data becomes available the additional capabilities of the computerized simulations may be used to investigate the possible correlations at high speeds.

(2) Bearing Kinematic Frequency Ω_k. A physical mechanism for the observed low frequency is not very clear at this stage but examining the X and R components somewhat closely, it seems evident that the motion is kinematic in nature. If it is assumed that the lower frequency response is indeed a sinusoidal motion with a fixed frequency then any such frequency should be related to the classical expression of the type:

$$\Omega = \frac{1}{2\pi}\sqrt{\frac{g}{l}} \tag{7}$$

where g is the relevant acceleration and l is the length of the oscillating pendulum.

Since the inner race is in static equilibrium with the applied thrust load, the axial component of the ball-race contact load may be assumed to provide an equivalent acceleration and since for low speed bearing the contact forces at the inner and outer races are equal the effective acceleration corresponding to oscillation of either raceway will be equal. However, the determination of the effective length is not quite straightforward. It can be assumed that the ball rolls on just one of the races and if it has to oscillate on both of the races then clearly the actual path of the ball is no longer circular. An additional complication is introduced by the fact that the inner race is moving axially in order to preserve the axial equilibrium. In the light of all these difficulties the effective length, l is strictly determined by examining the actual response and by assuming that the low frequency motion is indeed sinusoidal. The relevant kinematic frequency is thus defined as

$$\Omega_k = \frac{1}{2\pi}\sqrt{\frac{Q}{ml}} \tag{8}$$

where Q is the contact load and the effective length is determined numerically from the analytical simulation of the ball load.

The overall validity of equation (8) does not seem unreasonable

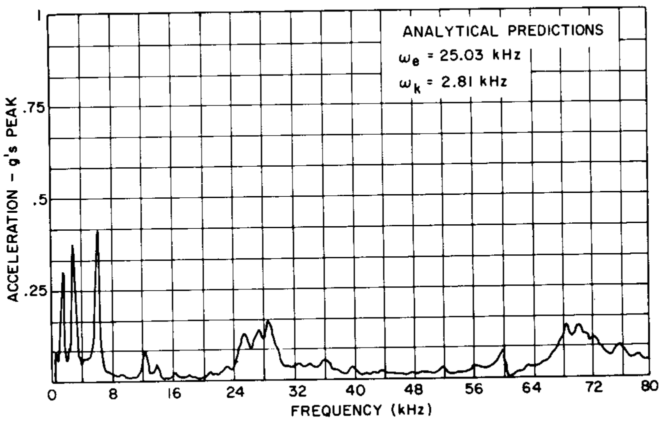

Fig. 5 Experimental frequency spectrum and comparison with analytical predictions for Bearing A

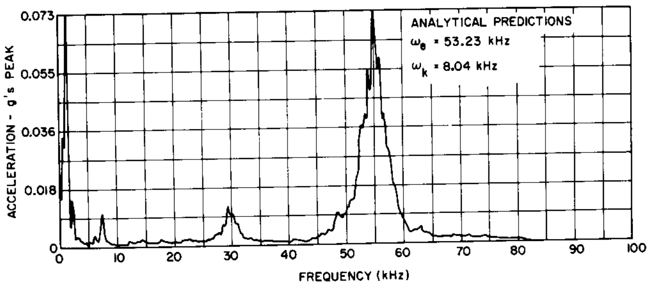

Fig. 6 Experimental frequency spectrum and comparison with analytical predictions for Bearing B

when the simulated frequency, ω_k is plotted as a function of load, Q in Fig. 4 as a ½ power variation is clearly noted. The value of l for the results shown in Fig. 4 is estimated to be 0.115 mm (4.53×10^{-3} in.).

Once again, since the available experimental data is in a relatively low speed region, the above correlations are intended only for low speed conditions.

Experimental Verification

In order to experimentally verify the existence of the above characteristic natural frequencies the analytical predictions are compared with the available experimental data for two different bearings. The data for one of the bearings, Bearing A used by Winn [3], is already given above. The second bearing is selected from the work of Broderick, et al. [1] and Meacher and Chen [2]. This bearing is denoted as Bearing B and the relevant bearing data is:

Ball diameter	= 7.9375 mm (0.3125 in.)
Pitch diameter	= 48.514 mm (1.9010 in.)
Contact angle	= 15°
Outer race curvature factor	= 0.53
Inner race curvature factor	= 0.515
Number of balls	= 15

The experimental data for both of these bearings is obtained by carrying out a conventional spectral analysis of a vibrational signal collected by an accelerometer mounted on the stationary outer race, when the inner race is subjected to a constant angular velocity of 8000 rpm under a prescribed thrust load of 4448 N (1000 lb). The details of the experimental set-up and associated electronics are closely identical to those of many similar investigations available in the literature and a reference to the work of Darlow and Badgley [4] will suffice for the salient features of the experimental apparatus.

Typical spectograms for Bearings A and B are shown in Figs. 5 and 6, respectively. The analytically simulated elastic contact and bearing kinematic frequencies are also stated on the spectograms. It is clear that the elastic contact frequency is the most dominant peak in the frequency spectrum and the existence of a peak corresponding to the kinematic frequency is also verified. Some of the other dominant frequencies present in the spectrum cannot be predicted by the simplified analytical simulation used in the present investigation.

There are additional factors which complicate the problem. Firstly, in a real system there is some mass associated with the moving race and this mass adds another degree-of-freedom in the vibratory motion. Secondly, the assumption that the low frequency motion is truly sinusoidal may not be realistic and, therefore, a Fourier decomposition of the true signal may result in more than one characteristic frequency. With all these uncertainties involved, the work presented here should be regarded only as a preliminary study at this stage. Perhaps, some more advanced computerized simulations, which the authors hope to provide in the near future, will reveal the additional greatly needed insight into the natural frequencies of a rolling bearing.

Conclusion

Pertinent to the vibratory motions in rolling bearings two characteristic natural frequencies have been defined and their existence is verified by both computerized analytical simulation and the available experimental data. An "elastic contact frequency," Ω_e, is correlated to a Hertzian spring relevant to the ball race contact. Although the physical model for the lower frequency component is not absolutely clear the motion is found to be basically kinematic in nature, and it is, therefore, termed as "bearing kinematic frequency," Ω_k. Primarily based on the physical properties associated with these two distinct modes, it is shown that Ω_e and Ω_k vary as respectively ⅙ and ½ powers of the ball-race contact loads. The entire investigation is restricted to only low speed bearings and the contact loads at the outer and inner race contacts are therefore closely identical. The existence of these two characteristic frequencies in the general ball motion in a bearing is also supported by the available experimental data.

Acknowledgments

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DISCUSSION

J. L. Farey[2]

I am very pleased to see analytical work being conducted in the area of high frequency vibration of ball bearings. Many investigators are successfully using this region of the bearing vibration spectrum to diagnose the condition of a ball bearing. A better understanding of the generation of high frequency vibration in ball bearings will aid all of us in the field.

I would however like to see new experimental work done in verifying the presence of the predicted vibration components. I do not have access to reference [3] of the paper, but have examined reference [1] which contains the data used for Fig. 6 of the paper. The experimental frequency spectrum is shown in Fig. F-1 of reference [1]. From this figure and the calculation of the paper, it would seem that the analytical/experimental agreement is excellent. If, however, one examines Fig. F-2 of reference [1], which is a vibration spectrum of the same bearing at 5 percent of the load, one would expect to see the signals at 55 KHz and 8 KHz to be reduced in frequency by the amount predicted in Fig. 4 of the paper. Fig. F-2 of reference [1], however, shows that the two signals have not shifted in frequency at all and remain at approximately 8 KHz and 55 KHz. It would seem therefore that these two signals are fixed resonances and not the Elastic Contact Frequency and the Bearing Kinematic Frequency signals as suggested in the paper.

Great care must be exercised in the high frequency region to eliminate transducer resonances. The signal at 55 KHz is particularly suspect in this regard. Its amplitude is almost directly proportional to bearing load which might be expected for an accelerometer resonance.

[2] Shaker Research Corp., Ballston Lake, N.Y.

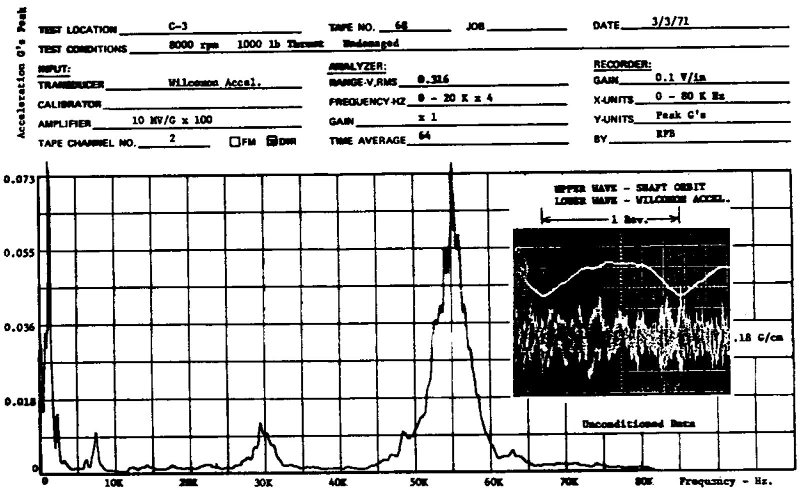

Fig. F-1 Radial acceleration spectrum, 1000 lb thrust, 8000 rpm undamaged

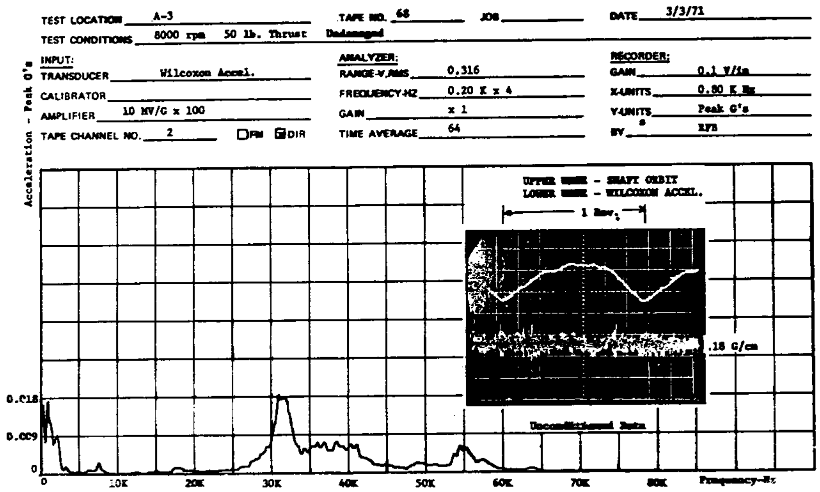

Fig. F-2 Radial Acceleration spectrum, 50 lb thrust, 8000 rpm, undamaged

O. G. Gustafsson[3]

The authors present very interesting results from their simulation technique to determine two heretofore unknown natural frequencies of ball bearings.

In their analysis, the authors study the vibration of the ball complement only and neglect the influence of the outer ring mass, while in the experiments the outer ring vibration was measured. This is understandable since including the effect of the outer ring mass and flexibility would make the analysis even more complex. The discussant feels that, at least in their first approach, the authors are justified in neglecting the effects of the outer ring and in using a frictionless model, since the two natural frequencies have been clearly identified under steady-state conditions in experimental vibration spectra.

In the discussant's laboratory, a series of outer ring resonances, different from those mentioned by the authors, were studied, considering the effects of outer ring mass and flexibility in its own plane.[4] The effect of ball mass was, however, neglected. The ball contacts were assumed to act as linear springs. The natural frequencies of the free outer ring (not mounted in a housing) are given by the equation

$$f_n = \frac{1}{2\pi}\left[\frac{(n^2-1)^2\dfrac{\pi EI}{R^3} + \dfrac{k_N Z}{2}}{\pi\rho_0 AR(1+1/n^2)}\right]^{1/2}$$

$$Z \geqq 2n + 1$$

where
f_n = natural frequency in Hz
E = Young's modulus of elasticity
I = second moment of area of the ring cross-section
R = mean radius of the ring
k_N = linearized Hertzian coefficient
ρ_0 = mass density of ring
A = cross-section area of ring
Z = number of balls
n = any integer > 0

The six lowest natural frequencies were computed for the authors' bearing A under the assumption that the ring and ball dimensions are the same as those of a 6210 bearing. These frequencies are 370, 472, 520, 607, 744, and 947 Hz. The lowest frequency, 370 Hz, represents rigid body motion of the outer ring, while the higher frequencies include the effects of outer ring bending. It is seen that in this case, all the computed resonant frequencies are too low to be observed in the authors' spectrum. Higher flexural natural frequencies also occur throughout the spectrum, but the amplitudes at these frequencies are generally too small to be detected. The outer ring resonances are highly influenced by the bearing dimensions. For a 6312 bearing, the computed three lowest natural frequencies are 2.49, 3.51, and 5.37 kHz. Fig. 7 shows an experimental spectrum of a 6312 bearing, which indicates fairly good agreement with the three computed frequencies.

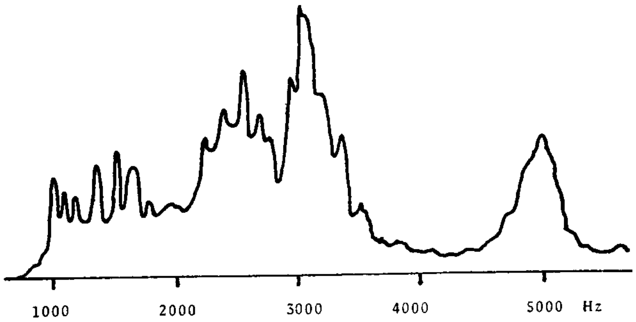

Fig. 7 Vibration spectrum of 6312 bearing from 1000 to 5000 Hz

[3] SKF Industries, Inc., King of Prussia, Pa.

[4] Gustafsson, O. G., Tallian, T. E., et al., "Final Report on the Study of the Vibration Characteristics of Bearings," U. S. Department of the Navy, Bureau of Ships, Contract NOb-78552, SKF Report AL63L023, DDC AD 432 979, 1963.

Authors' Closure

The authors agree with Mr. Frarey in the fact that more experimental data is needed in order to understand the dynamic characteristics of rolling bearings. It is true that a number of frequencies, other than the two defined by the authors, are present in any experimental spectrum. The precise load dependence of the various frequencies is also not known and the authors agree that some of the frequencies could indeed be independent of load or other operating conditions. In view of all these uncertainties the objective of this paper is only to show that the computed characteristic frequencies do indeed exist in experimental frequency spectra discussed in this paper.

At a light load of 50 lb, in the case of Bearing B, the computed elastic contact and kinematic frequencies will respectively reduce to about 32.2 kHz and 1.80 kHz. The existence of these frequencies in the spectrum shown in Fig. F-2, by Mr. Frarey, cannot be completely denied. The fact that relatively small peaks continue to exist at about 8 kHz and 55 kHz, once again suggests a more extensive analytical and experimental investigation aimed at a systematic characterization of all dominant frequencies. The authors agree that it is important to eliminate transducer and other resonances associated with the experimental apparatus.

Bending of the outer race will certainly contribute to several characteristic peaks in the very low frequency region as pointed out by Mr. Gustafsson. The authors look forward to a more complete dynamic simulation which to some extent will allow for the flexibility of the races.