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Viscoelastic Effects in MIL-L-7808-Type Lubricant, Part I: Analytical Formulation©

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Analytical formulations for the computation of lubricant film thickness and traction in a high-speed rolling-sliding contact are presented with the objective of investigating the viscoelastic response of the MIL-L-7808-type lubricant. Two types of relations are used to model the viscous shear strain rate. In the Type I model, a hyperbolic sine relation is used to model the viscous effect which becomes significant when the shear stress reaches a critical value. The Type II model employs a limiting shear stress, which the lubricant can withstand, and an inverse hyperbolic tangent function is considered to model the viscous behavior. Both models are based on three fundamental properties: lubricant viscosity, shear modulus and a critical shear stress. While the viscosity relations may be obtained by direct measurements, estimates of shear modulus and critical shear stress may be derived by curve-fitting the model predictions to experimental traction data.

KEY WORDS

Traction, Viscosity, Thermal Reduction Factor, Film Thickness, Traction Modeling

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INTRODUCTION

The frictional behavior between interacting mechanical elements plays a dominant role in the overall performance and life of intricate mechanical components, such as rolling bearings, gears and transmissions. It is well known that proper lubrication prevents excessive metallic contacts, reduces friction and wear of interacting elements, and often serves as an effective cooling media. While the design of a lubrication system assures the flow of lubricant to the contact interface between mating elements, the rheological behavior of the lubricant in the contact zone determines the frictional stability of the contact and the overall performance of the component or the entire mechanical system. For example, in rolling bearings, the tractive forces at the rolling element/race interface greatly influence the orbital acceleration of rolling elements which, in turn, determines the extent of rolling element/cage collisions and associated cage instabilities. Thus, cage instabilities, ball and roller skid, roller skew and excessive wear associated with the highly dynamic interaction between the bearing elements are often attributed to the traction behavior of the lubricant.

Over the past three decades, a significant fraction of the mechanical engineering literature has been devoted to elastohydrodynamic lubrication and lubricant rheology. Investigations have ranged from the most fundamental prob-

NOMENCLATURE

a = major half-width of contact, M
b = minor half-width of contact, M
E_1 = elastic modulus of interacting body 1, Pa
E_2 = elastic modulus of interacting body 2, Pa
G = shear modulus, Pa
h = lubricant film thickness, M
h_{iso} = isothermal film thickness, M
K_f = thermal conductivity, $N/S/K$
p_o = Hertz contact pressure, Pa
Q = applied load, N
R_1 = radius of body 1 in rolling direction, M
R_2 = radius of body 2 in rolling direction, M
$\dot{s}$ = shear strain rate, $1/S$
T = temperature, K
T_o = reference temperature, K
U_1 = Rolling velocity of body 1, M/S
U_2 = rolling velocity of body 2, M/S
α = pressure viscosity coefficient, $1/Pa$
β = temperature-viscosity coefficient, $1/K$ of K
γ = pressure-temperature-viscosity coefficient, $1/K/Pa$ or K/Pa
κ_m = maximum traction coefficient
μ = viscosity at any pressure and temperature $Pa.S$
μ_o = reference viscosity, $Pa.S$
ν_1 = Poisson's ratio for body 1
ν_2 = Poisson's ratio for body 2
τ = shear stress, Pa
τ_o = critical shear stress, Pa

lem of lubricant behavior at a molecular level to very applied problems where the global hydrodynamics intricately couples with the elastic distortion of the load bearing contacts in a rolling bearing. The rolling-disk type of test rigs have proven to be very effective for the evaluation of the tractive behavior of a lubricant. The most commonly used military oil, with a specification of MIL-L-7808 has been tested by a number of investigators. However, due to both the complexities in lubricant behavior and the experimental difficulties in traction measurement, the range of operation over which the data have been obtained is limited. In an attempt to widen this data range, traction experiments with a number of different specimens,providing a range of contact ellipticity ratios, are undertaken in the present investigation. Also, particular emphasis is given to rolling velocities higher than those considered in the past.

Analytical development for understanding lubricant rheology and modeling traction behavior in concentrated contacts has been a subject of many researchers. The amount of available literature is rather vast, and a fair review is beyond the limits of this paper. Both Newtonian and non-Newtonian types of rheological behaviors have been investigated. The experimentally observed traction/slip slopes at low sliding velocities, and an asymptotic nature of traction at very high sliding velocities have prompted the investigations based on viscoelastic behavior. Johnson and Tevaarwerk (*1*) have proposed an Eyring sinh law as a nonlinear viscous function for a number of lubricants. The constitutive relation is expressed in terms of three fundamental parameters: the shear modulus, the zero rate viscosity and a reference shear stress. A slightly different relation which modifies the classical Maxwell equation by a limiting shear stress has been proposed by Bair and Winer (*2*). Again, the constitutive equation is defined by three fundamental properties: low shear stress viscosity, limiting shear modulus, and the limiting shear stress which the fluid can withstand. Based on these two common models, the objective of the present investigation is to evaluate the viscoelastic effects in the MIL-L-7808 type lubricant.

Direct measurement of the required fundamental properties of the lubricant is, indeed, very difficult. Correlations of the traction behavior to the fundamental properties of the lubricant require that the rheological behavior be measured at very high pressures and carefully controlled temperatures. Although viscosity measurements as a function of pressure and temperature have been possible (*3*), direct measurement of shear modulus and critical shear stress has been far too complex. An acceptable approach has been to estimate these constitutive constants from experimental traction data obtained with a rolling-disk type of traction apparatus (*4*). A given model is basically curve-fitted to the experimental data to estimate the rheological constants. It is quite true that since the overall measured traction includes significant effects, such as tangential deformation of the disk specimens, such an approach may not provide the true values of fundamental properties, such as shear modulus or critical shear stress. However, the curve-fitted values may be acceptably used as effective values for traction predictions when the data correlation is good. Such a hypothesis is the foundation of the present approach.

In addition to the lubricant constitutive equation, an important input to most of the traction models discussed above is the lubricant film thickness in the contact. Again, the available literature is too vast to be reviewed here. For the computation of isothermal film thickness in elliptical contacts, the formulae presented by Hamrock and Dowson (*5*) are well accepted. It is also well understood that the isothermal conditions are only valid under relatively low speeds. At high rolling speeds, and also at high slip velocities, the film thickness must be corrected for thermal effects. Perhaps the first thermal solution to the line contact problem was developed by Cheng (*6*), where not only the viscous shear heating effects and heat transfer in the lubricant film were modeled but also the heat conducted to the moving surfaces was taken into account. Subsequent to this work, there have been a number of investigations aimed at modeling the thermal effects in concentrated rolling/sliding contacts. The formulation used in the present investigation is based on the work by Cheng (*6*) and by Wilson and Sheu (*7*).

This paper is the first part of an overall investigation and it presents the analytical formulation of the models used. Experimental procedures and data correlations are subjects of the second paper, and the application of the traction model to a rolling bearing will be the subject of the third paper.

Analytical formulation of a traction model may be divided into three basic parts: viscosity-pressure-temperature relations, computation of lubricant film thickness, and the development of traction-slip relation in a concentrated contact. A discussion of the formulations used in these various analytical steps is the subject of this paper.

VISCOSITY RELATIONS

Characterization of lubricant viscosity behavior is a first step in the development of a traction model. The general approach is to measure the viscosity at prescribed pressures and temperatures and then fit empirical relations to the data. Analytical expressions used to describe the viscosity-pressure-temperature behavior are commonly of two types. For the present investigation, these relations are referred to as Type I and Type II.

The first type of viscosity relation, Type I, simply permits an exponential variation of viscosity as a function of pressure and temperature. When there is a coupling between pressure and temperature, an additional term may be added to represent such a coupling. With the various symbols described in the nomenclature, the viscosity is written as:

$$\mu = \mu_o \exp\{\alpha p + (\beta + \gamma p)(T_o - T)\} \qquad [1]$$

The Type II relation is quite similar to Eq. [1], except that the temperature variation is expressed as the inverse of absolute temperature. The viscosity relation is written as:

$$\mu = \mu_o \exp\left\{\alpha p + (\beta + \gamma p)\left(\frac{1}{T} - \frac{1}{T_o}\right)\right\} \qquad [2]$$

It is clear that with increasing temperature, the viscosity approaches an asymptotic value with the Type II relation, while it continues to decrease exponentially with the Type I relation. Aside from this observation, there is no real difference between the two relations.

LUBRICANT FILM THICKNESS MODELING

Modeling of lubricant film thickness in an elastohydrodynamic contact is accomplished in two steps. First, the film profile is computed by solving the coupled problem associated with the elastic distortion of the interacting surfaces and the fluid hydrodynamics. Then, a correction to account for thermal effects is applied to derive the final film thickness in the contact. In view of the very high surface finish of the test specimens and the flooding of the contact with lubricant jets, as discussed in the second part of this paper, the surface roughness and starvation effects on the lubricant film thickness are not considered in the present investigation. The ratio of lubricant film thickness to surface finish ranges from 7 to 20. As shown schematically in Fig. 1, the overall shape of the lubricant film is well established within the current theory of elastohydrodynamic lubrication. The film thickness is fairly constant over most of the contact zone, and the well defined dip near the exit region determines the minimum film thickness. Based on the complete numerical solutions, formulae for the computation of both the nominal and minimum film thickness are available. While the minimum film thickness is of significance in partial elastohydrodynamic lubrication where direct interaction between surface asperities may occur, the nominal, or central, film thickness holds for most of the contact zone. It is, therefore, the primary input to the computation of traction in well-lubricated elastohydrodynamic contacts. Emphasis is therefore placed on the computation of nominal film thickness in the present investigation.

The isothermal film thickness, h_{iso}, in the present investigation is computed by the Hamrock and Dowson (*5*) formula, which is written as:

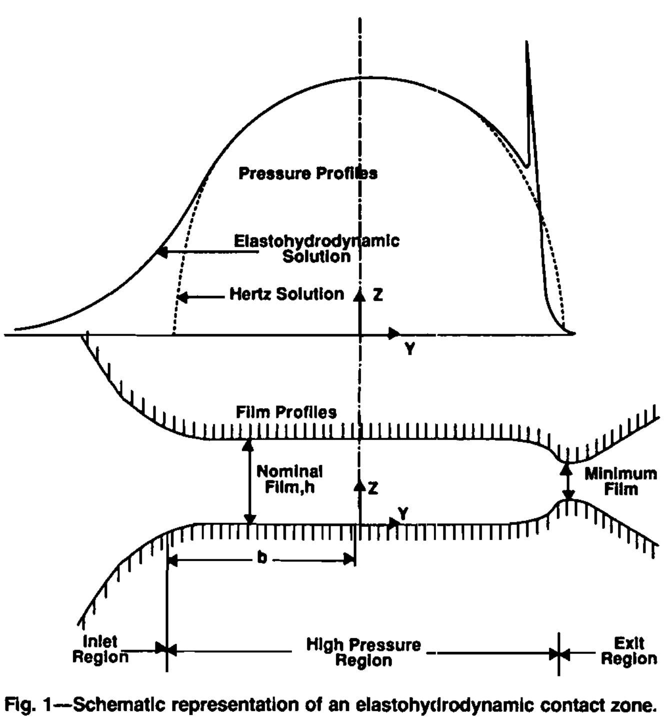

Fig. 1—Schematic representation of an elastohydrodynamic contact zone.

$$\frac{h_{iso}}{R} = 2.69 \frac{\overline{U}^{0.67} G^{0.53} (1 - 0.61 e^{-0.73k})}{W^{0.067}} \qquad [3]$$

where the various dimensionless parameters are defined, in terms of the fundamental variables included in the nomenclature, as:

$$\overline{U} = \frac{\mu_o U}{E'R}\text{, the speed parameter}$$

$$U = \frac{U_1 + U_2}{2}\text{, the rolling speed}$$

$$\frac{1}{E'} = \frac{1}{2}\left\{\frac{1 - \nu_2}{E_1} + \frac{1 - \nu_2}{E_2}\right\}\text{, the materials parameter}$$

$$R = \frac{R_1 R_2}{R_1 \pm R_2}\text{, the equivalent radius}$$

$$G = \alpha E'\text{, the elasticity parameter}$$

$$W = \frac{Q}{E'R}\text{, the load parameter}$$

$$k = \frac{a}{b}\text{, the ellipticity parameter}$$

The isothermal film thickness computed above is modified by a thermal reduction factor, Φ_T, to derive the final value of film thickness, h, in the contact under prescribed operating conditions.

$$h = \Phi_T\, h_{iso} \qquad [4]$$

Modeling of thermal effects is based on the work by Cheng (*6*) and Wilson and Sheu (*7*). An attempt is made to modify the Wilson and Sheu formula to account for load effects as done by Cheng. This resulted in considerable computer analysis of both models, and empirical curve-fitting of the numerical solutions. The result is the following equation for the thermal reduction factor:

$$\Phi_T = \frac{1 - 13.2 \left(\frac{p_o}{E'}\right) L^{0.42}}{1 + 0.213\,(1 + 2.23\, S^{0.83})\, L^{0.640}} \qquad [5]$$

where the thermal loading parameter, L, and the slide-to-roll ratio, S, are defined as

$$L = \left(-\frac{\partial\mu}{\partial T}\right)\frac{U^2}{K_f} \text{ and } S = 2\,\frac{U_1 - U_2}{U_1 + U_2}$$

The effect of contact load is indicated by the term containing the Hertzian pressure p_o. It is found that this effect is only significant at high values of slide-to-roll ratios and the thermal loading parameter. Figure 2 compares the variation in thermal reduction factors as computed by the above equation and those obtained by the original Wilson and Sheu formula.

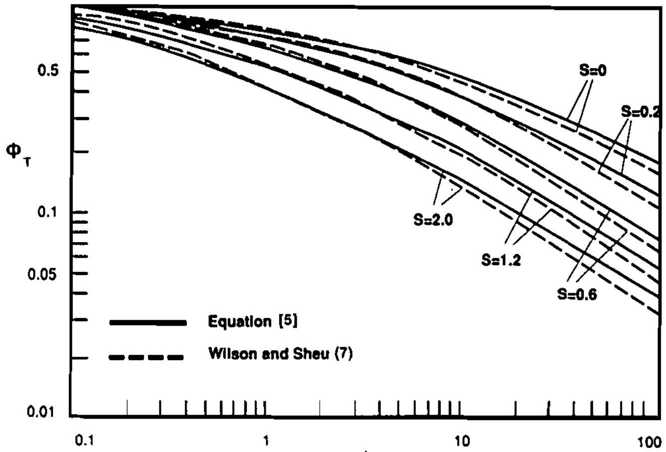

Fig. 2—Comparison of thermal reduction factors as predicted by Eq. [5] and the Wilson and Sheu formula (7).

TRACTION MODELS

The lubricant viscosity relations and the formulae for the computation of lubricant film thickness in a contact are two of the three inputs required in the prediction of traction in an elastohydrodynamic contact. The shear stress and strain rate constitutive equation, is the final input which completes the requirements of a traction model. It is the nature of this relationship which determines the type of a model. A common form of the shear stress and strain rate equation for a viscoelastic model is written as (*1*):

$$\dot{s} = \frac{1}{G}\frac{\partial \tau}{\partial t} + \frac{\tau_o}{\mu} f\left(\frac{\tau}{\tau_o}\right) \qquad [6]$$

In the above equation, the three fundamental properties, G, τ_o, and μ, are respectively, the shear modulus, critical shear stress and lubricant viscosity. All these properties are, in general, functions of pressure and temperature.

When the rolling velocity is U, the convective derivative term, as explained by Johnson and Tevaarwerk (*1*) can be replaced as shown:

$$\frac{\partial}{\partial t} = U\frac{\partial}{\partial y}$$

where y is the coordinate along the rolling direction, as shown in Fig. 1.

Also, when the sliding velocity is U_s, lubricant film thickness is h, and the contact half width is b, the following quantities may be defined to express Eq. [6] in a more convenient form:

$$\dot{s} = \frac{U_s}{h}\ \bar{\tau} = \frac{\tau}{\tau_o}\ \bar{y} = \frac{y}{b}$$

Equation [6] may now be written as:

$$\frac{\partial \bar{\tau}}{\partial \bar{y}} = \dot{\bar{s}}\ - \frac{1}{D}f(\bar{\tau}) \qquad [7]$$

where

$$D = \frac{\mu U}{Gb}, \text{ the Deborah Number}$$

$$\dot{\bar{s}} = \frac{U_s Gb}{hU\tau_o}, \text{ the dimensionless strain rate.}$$

Similar to the Newtonian model, the functional relationship, $f(\bar{\tau})$, in the above equation defines the type of a viscoelastic model. The two common types of this function are:

Type I:

$f(\bar{\tau})$ = sinh ($\bar{\tau}$), the Johnson and Tevaarwerk model (*1*)

Type II:

$f(\bar{\tau}) = \tanh^{-1}(\bar{\tau})$, the Bair and Winer model (*2*)

Aside from the above functional relationship, certain assumptions about the pressure and temperature distribution in the contact are essential before Eq. [7] may be applied to a contact. It is generally reasonable to assume that the pressure conforms to the Hertzian contact solution. The computation of temperature distribution, however, is a more complex task. Generally, the interacting surfaces have to be modeled as a moving heat source, and the heat transfer equations have to be solved to obtain a temperature distribution along the rolling direction for heat generated in the contact due to lubricant shear. For the present investigation, a simplified assumption that the interacting surfaces maintain a constant temperature, T_o, is made, and Eq. [7] is applied to the central plane through the film where the temperature rise is simply computed by conduction through the film for the given heat generated at any point in the contact:

$$\Delta T = \frac{\tau U_s h}{4\ K_f} \qquad [8]$$

It is, of course, assumed here that the strain rate resulting from the sliding velocity, U_s, is in the viscous regime.

For a given pressure and temperature, the fundamental properties, G, τ_o and μ, may be defined at any point in the contact, and Eq. [7] can then be applied to compute traction. When the slip is completely in the rolling direction, as is the case in the rolling disk apparatus, Eq. [7] is essentially integrated numerically over the range $-1 \leq \bar{y} \leq 1$ with the initial condition:

$$\text{at } \bar{y} = -1, \bar{\tau} = 0.$$

The computed shear stress may then be integrated over the contact ellipse to determine the total traction.

Effective values for the fundamental properties, shear modulus, critical shear stress and lubricant viscosity are derived by curve-fitting the model predictions to the experimental data. While estimates of viscosity may be obtained directly from the viscosity measurements and from Eqs. [1] and [2], the effective shear modulus and critical shear stress are back-calculated by fitting the model to measured traction data. As a first step to such a curve fitting, or regression analysis, both the shear modulus and critical shear stress are assumed to have a constant value over the entire contact area for a given operating environment. Under such simplified assumptions, Figs. 3 and 4 show typical variation of traction as a function of the shear modulus G, and the critical shear stress, τ_o, respectively for the Type I viscoelastic function. For a relatively high Deborah number, the behavior is essentially elastic at very small strain rates (i.e., very low slip velocities). Thus, by neglecting the viscous term, assuming a constant average value for the shear modulus, and carrying out straightforward algebra, it can be shown that the slope of the traction curve, tan θ, is (*4*):

$$\tan \theta = \frac{4Gb}{\pi p_o h} \qquad [9]$$

Thus, the effective shear modulus is directly proportional to the slope of the traction curve.

As the slip increases, the viscous term becomes significant, and at high values of slip, the behavior is basically viscous. The traction coefficient in the purely viscous region increases very slowly with slip rate. In fact, as seen in Fig. 4, the traction may be assumed to be practically constant. If the Type I constitutive equation is applied under purely viscous conditions, the algebraic relationship between the traction coefficient, κ_m, at a fairly large slide-to-roll ratio, $(U_s/U)_m$, may be shown to be:

$$\left(\frac{U_s}{U}\right)_m \left(\frac{U\bar{\mu}}{hp_o}\right) = \left(\frac{\tau_o}{p_o}\right) \exp\left\{\frac{2}{3}\frac{\kappa_m p_o}{\tau_o}\right\} \qquad [10]$$

where $\bar{\mu}$, is the average viscosity over the contact.

Thus, for any given maximum traction coefficient, at a relatively large slide-to-roll ratio, the average value of the critical shear stress can be readily determined by solving the above algebraic equation.

The behavior of traction with the Type II viscoelastic function is quite similar to that seen above for the Type I model. At low slip rates, the behavior is purely elastic and Eq. [9] is valid for high Deborah numbers. At high slip rates, however, as seen in Fig. 5, traction approaches a constant value which is defined by a limiting shear stress. Thus the critical shear stress, τ_o, is actually a limiting shear stress which the material can withstand (*2*). Under such a condition, the maximum or limiting traction coefficient may be simply written as:

$$\kappa_m = \frac{3}{2}\frac{\tau_o}{p_o} \qquad [11]$$

Thus, for any given value of maximum traction, the effective value of the limiting shear stress can be readily determined.

For a given traction curve, the above equations provide preliminary values of G and τ_o. An adjustment to these values is now applied while minimizing the squared deviation between the predicted traction and the experimental data obtained over a range of operating conditions. Such a regression analysis also defines the variation of shear modulus and critical shear stress as a function of pressure and temperature.

A discussion of the results and final conclusions is deferred until the presentation of the experimental part of this investigation, which is the subject of the second part of this paper.

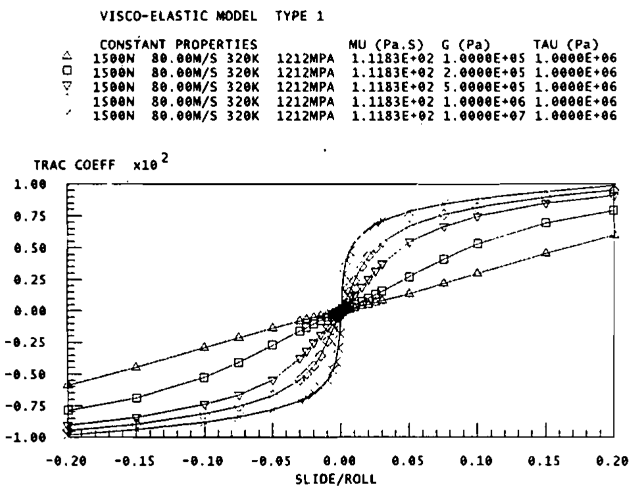

Fig. 3—Prediction of Type I viscoelastic model as a function of shear modulus.

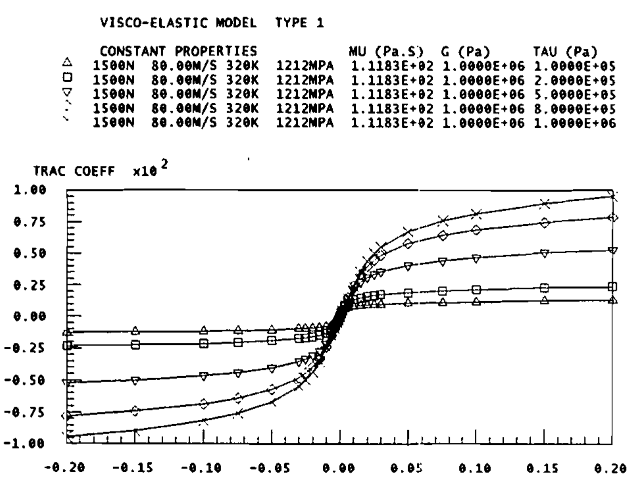

Fig. 4—Traction prediction of Type I viscoelastic model as a function of critical shear stress.

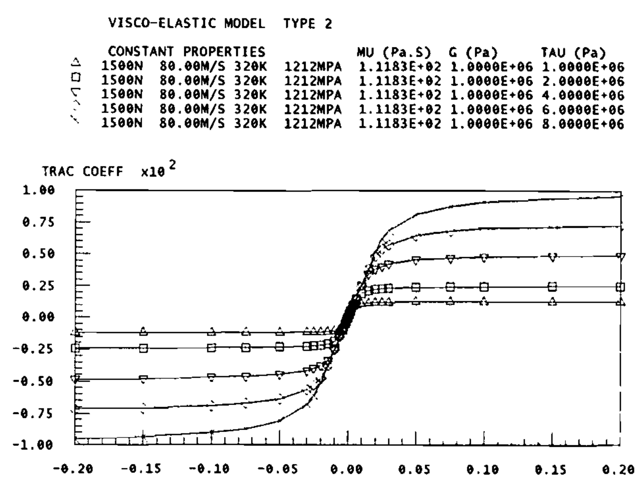

Fig. 5—Traction prediction of Type II viscoelastic model as a function of the limiting shear stress.

SUMMARY

Analytical formulation for viscoelastic traction is presented in three parts: viscosity-pressure-temperature relations, computation of lubricant film thickness, and shear stress and strain rate relations. Two types of relations are used to model the viscous shear strain rate. In the Type I model, a hyperbolic sine relation is used to model the viscous effect which becomes significant when the shear stress reaches a critical value. The Type II model employs a limiting shear stress, which the lubricant can withstand, and an inverse hyperbolic tangent function is considered to model the viscous behavior. Both models basically employ three fundamental quantities, i.e., lubricant viscosity, shear modulus, and critical shear stress. While the viscosity behavior may be derived from direct measurements, estimates of average shear modulus and critical shear may be obtained by curve-fitting the model predictions to experimental traction data.

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